Probability Review and Counting Fundamentals

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Overview

- Probability Review
- Fundamentals of Counting
  - Permutations: *ordered arrangements*
  - Combinations: *unordered arrangements*
- Selected Activities

Probability Review

- Definitions
- Classical Probability
- Relative Frequency Probability
- Probability Fundamentals and Probability Rules

What is Probability?

- Probability

  the study of chance associated with the occurrence of events

- Types of Probability
  - Classical (Theoretical)
  - Relative Frequency (Experimental)

Classical Probability

Rolling dice and tossing a coin are activities associated with a classical approach to probability. In these cases, you can list all the possible outcomes of an experiment and determine the actual probabilities of each outcome.

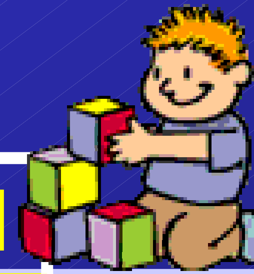
Listing **All Possible Outcomes** of a Probabilistic Experiment

- There are various ways to list all possible outcomes of an experiment
  - Enumeration
  - Tree diagrams
  - Additional methods – counting fundamentals

Three Children Example

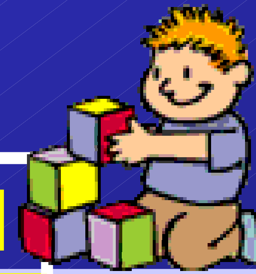
- A couple wants to have exactly 3 children. Assume that each child is either a boy or a girl and that each is a single birth.
- List all possible orderings for the three children.

Enumeration



1st Child	2nd Child	3rd Child

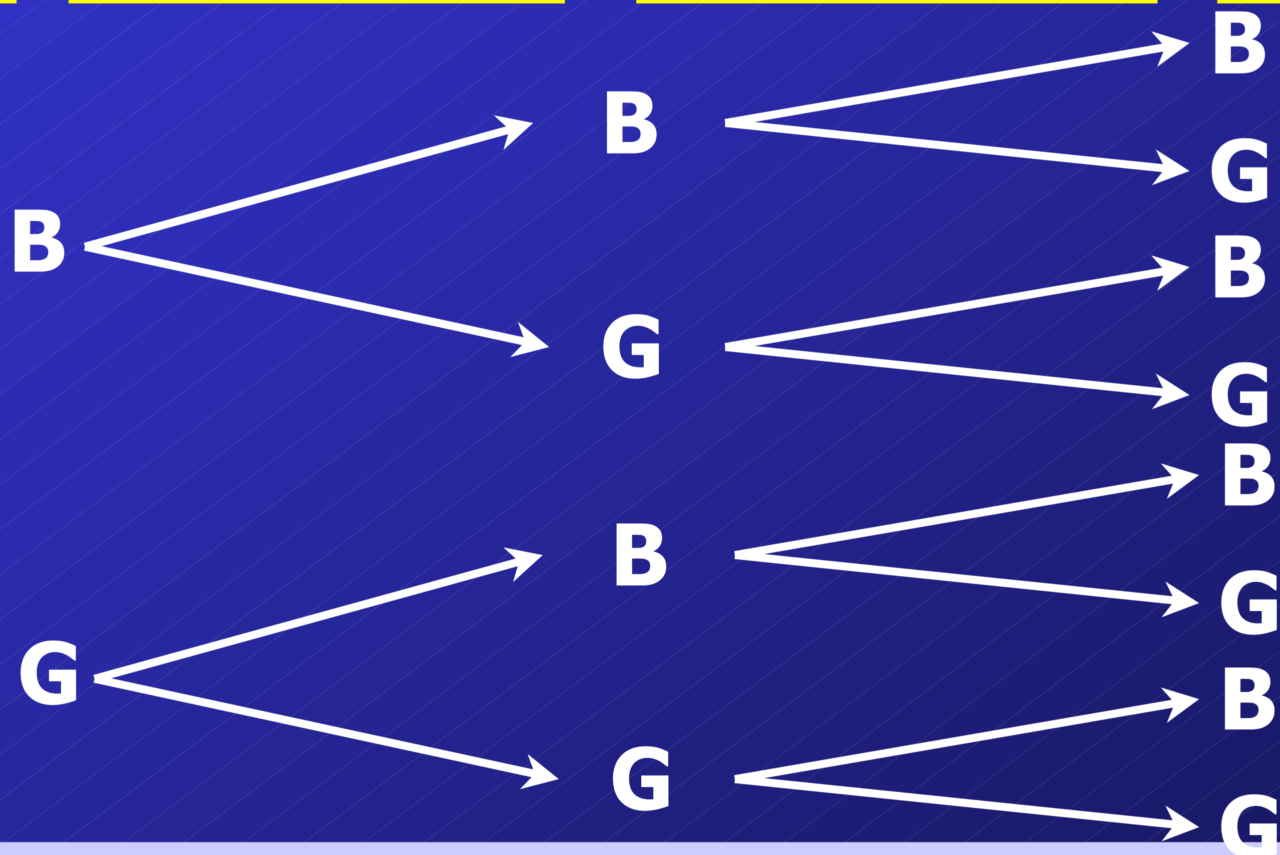
Enumeration



1st Child	2nd Child	3rd Child
B	B	B
G	B	B
B	G	B
B	B	G
G	G	B
G	B	G
B	G	G
G	G	G

Tree Diagrams

1st Child **2nd Child** **3rd Child**



BBB

BBG

BGB

BGG

GBB

GBG

GGB

GGG

Definitions

- **Sample Space** - the list of all possible outcomes from a probabilistic experiment.
  - 3-Children Example:
    S = {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG}
- Each individual item in the list is called a **Simple Event** or **Single Event.**

Probability Notation

P(*event*) = Probability of the *event* occurring

Example: P(Boy) = P(B)=½

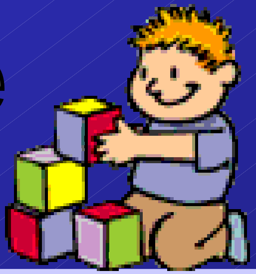
Probability of Single Events with Equally Likely Outcomes

- If each outcome in the sample space is equally likely, then **the probability of any one outcome is 1 divided by the total number of outcomes**.

For equally likely outcomes,

$$P(\text{simple event}) = \frac{1}{\text{total number of outcomes}}$$

Three Children Example Continued



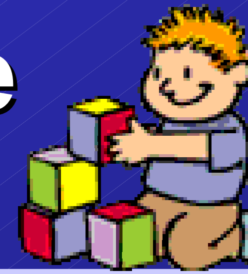
- A couple wants 3 children. Assume the chance of a boy or girl is **<u>equally likely</u>** at each birth.
- What is the probability that they will have exactly 3 girls?
- What is the probability of having exactly 3 boys?

Probability of Combinations of Single Events

- An event can be a combination of *Single Events*.

- The probability of such an event is the sum of the individual probabilities.

Three Children Example Continued



P(exactly 2 girls) = ___

P(exactly 2 boys) = ___

P(at least 2 boys) = ___

P(at most 2 boys) = ___

P(at least 1 girl) = ___

P(at most 1 girl) = ___

- Sample space =

Types of Probability

- Classical (Theoretical)
- Relative Frequency (Experimental, Empirical)

Relative Frequency Probability

- Uses actual experience to determine the likelihood of an outcome.

- What is the chance of making a B or better?

Grade	Frequency
A	20
B	30
C	40
Below C	10

Relative Frequency Probability is Great Fun for Teaching

- Rolling Dice
- Flipping Coins
- Drawing from Bags without Looking (i.e. Sampling)
- Sampling with M&M's (http://mms.com/cai/mms/faq.html#what_percent)

Empirical Probability

- Given a frequency distribution, the probability of an event, E, being in a given group is

$$P(\mathrm{E}) = \frac{\text{frequency of the group}}{\text{total frequencies in the distribution}} = \frac{x}{n}$$

Two-way Tables and Probability

	Made A	Made < A	Total
Male	30	45	
Female	60	65	
Total			

- Find

P(M)

P(A)

P(A and M)

Teaching Idea

- Question: How Can You Win at Wheel of Fortune?
- Answer: Use Relative Frequency Probability (see handout)

Source. Krulik and Rudnick. “Teaching Middle School Mathematics Activities, Materials and Problems.” p. 161. Allyn & Bacon, Boston. 2000.

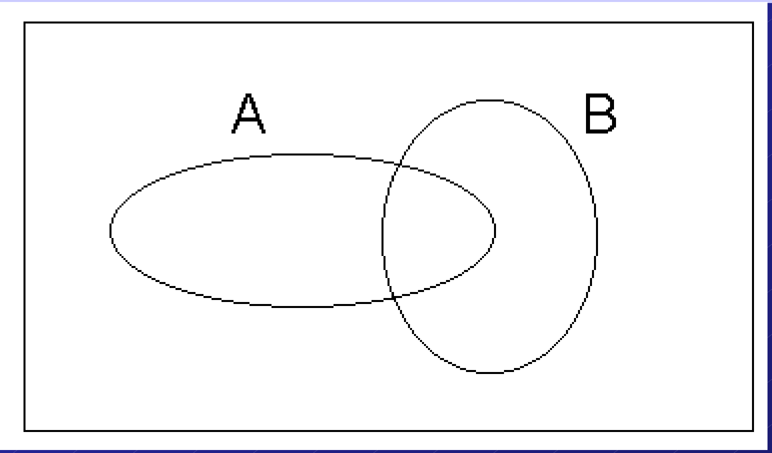
Probability Fundamentals

- What is wrong with the statements?
  - The probability of rain today is -10%.
  - The probability of rain today is 120%.
  - The probability of rain or no rain today is 90%.

$$P(event) \geq 0$$
$$P(event) \leq 1$$
$$P(sample\ space) = 1$$

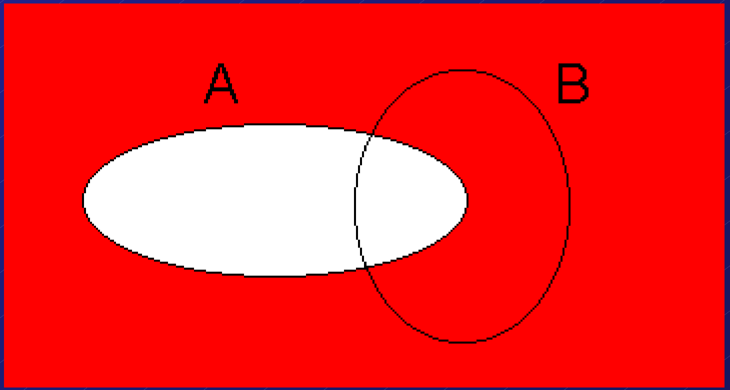
Probability Rules

Let A and B be events



Complement Rule:

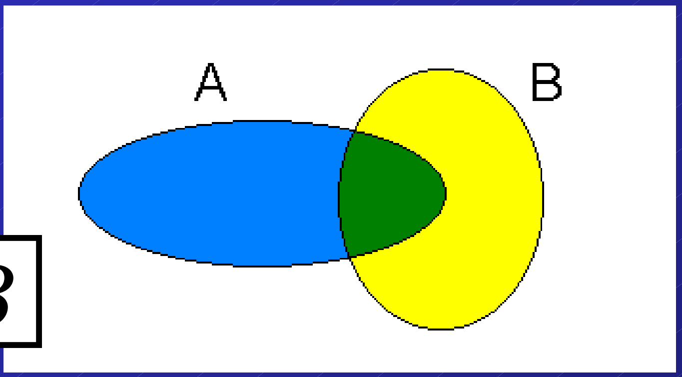
P(A) + P(not A) = 1



Set Notation

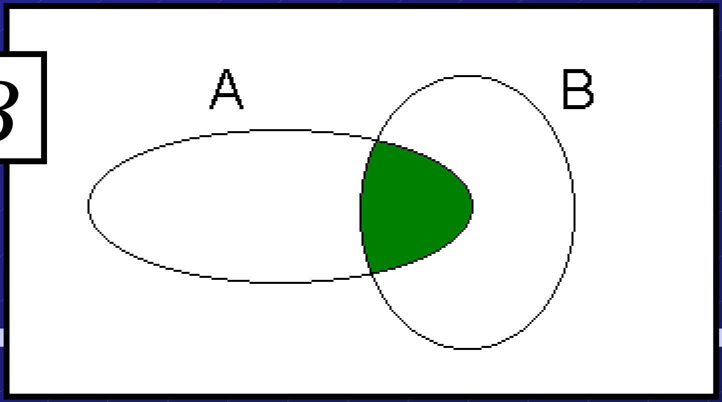
- Union: A or B (inclusive “or”)

$$A \cup B$$



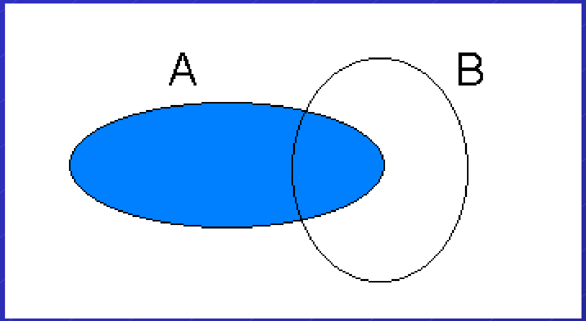
- Intersection: A and B

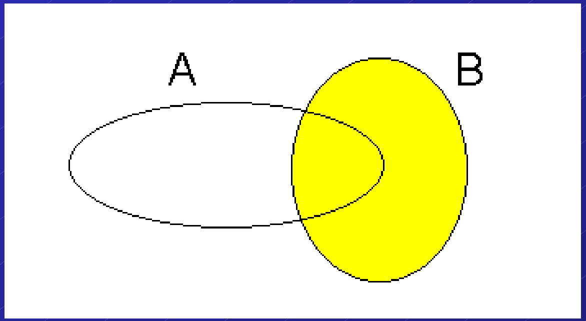
$$A \cap B$$

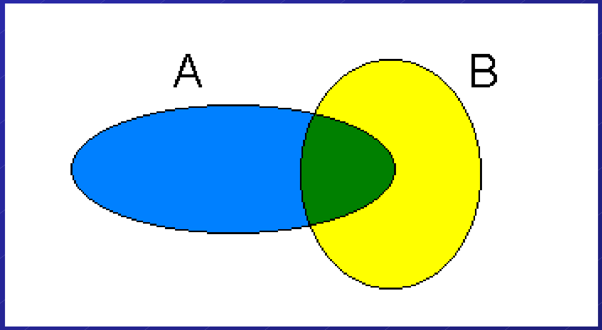


Probability Rules

Union   P(AUB) = P(A or B)







$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Teaching Idea

- Venn Diagrams
- Kyle Siegrist's Venn Diagram Applet

http://www.math.uah.edu/stat/applets/index.xml

Two-way Tables and Probability

	Made A	Made < A	Total
Male	30	45	75
Female	60	65	125
Total	90	110	200

- Find

P(M)

P(A)

P(A and M)

P(A if M)

Conditional Probability

P(A|B) = the conditional probability of event A happening given that event B has happened

"probability of A given B"

$$\boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)}}$$

Independence

- Events A and B are "Independent" if and only if

$$\boxed{P(A \mid B) = P(A)}$$

- From the two-way table, is making an "A" independent from being male?

Teaching Idea: Discovery Worksheets

- Basic Probability Rules (see handout)
- Basic Probability Rules (long version) http://www.mathspace.com/NSF_ProbStat/Teaching_Materials/Lunsford/Basic_Prob_Rules_Sp03.pdf
- Conditional Probability http://www.mathspace.com/NSF_ProbStat/Teaching_Materials/Lunsford/Conditional_Prob_Sp03.pdf

Overview

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  - Permutations: *ordered arrangements*
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Counting Techniques

- Fundamentals of Counting
- Permutations: *ordered arrangements*

- Combinations: *unordered arrangements*

Fundamentals of Counting

- *Q*: Jill has 9 shirts and 4 pairs of pants. How many different outfits does she have?
- *A*:

Fundamentals of Counting

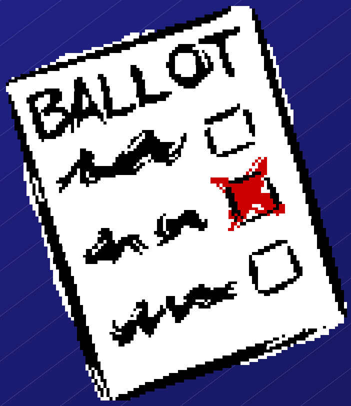
- Multiplication Principle:

If there are ***a*** ways of choosing one thing, and ***b*** ways of choosing a second thing after the first is chosen, then the total number of choice patterns is:

a* x *b

Fundamentals of Counting

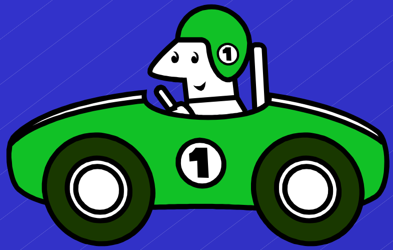
- *Q*: 3 freshman, 4 sophomores, 5 juniors, and 2 seniors are running for SGA representative. One individual will be selected from each class. How many different representative orderings are possible?



- A:

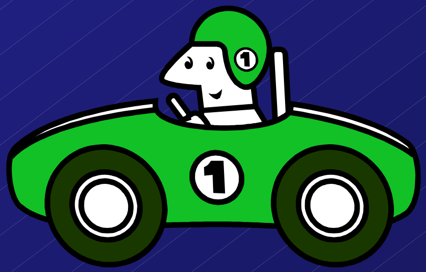
Fundamentals of Counting

- **Generalized Multiplication Principle**:
- If there are ***a*** ways of choosing one thing, ***b*** ways of choosing a second thing after the first is chosen, and ***c*** ways of choosing a third thing after the first two have been chosen…and ***z*** ways of choosing the last item after the earlier choices, then the total number of choice patterns is $a \times b \times c \times \ldots \times z$



Local Examples

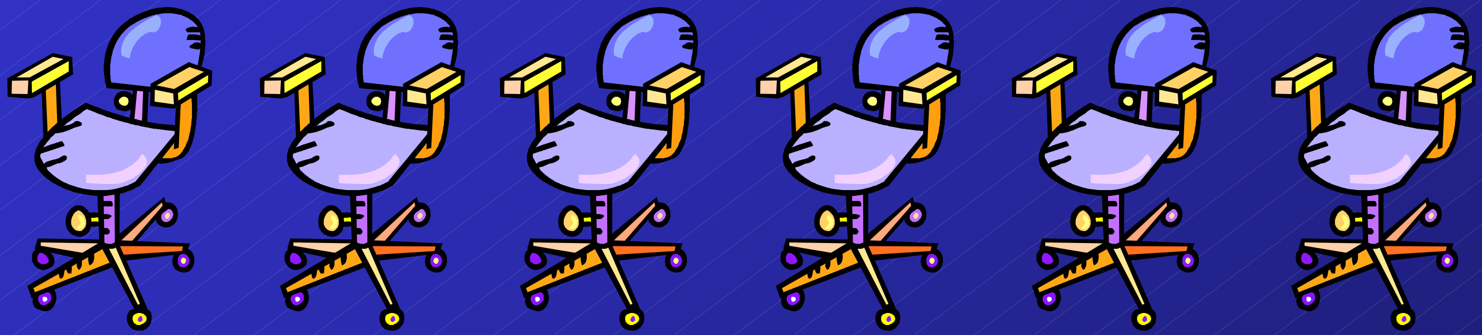
- Q: When I lived in Madison Co., AL, the license plates had 2 fixed numbers, 2 variable letters and 3 variable numbers. How many different license plates were possible?
- A:



Fundamentals of Counting

- Q: How many more license plate numbers will Madison County gain by changing to 3 letters and 2 numbers?
- A:

Permutations: Ordered Arrangements



- Q: Given 6 people and 6 chairs in a line, how many seating arrangements (orderings) are possible?
- A:

Permutations: Ordered Arrangements

- Q: Given 6 people and 4 chairs in a line, how many different orderings are possible?
- A:

Permutations: Ordered Arrangements

- Permutation of *n* objects taken *r* at a time:

  r-permutation, $P(n,r)$, ${}_nP_r$

- Q: Given 6 people and 5 chairs in a line, how many different orderings are possible?

- A:

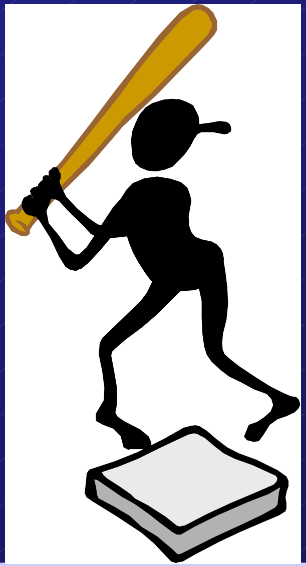
Permutations: Ordered Arrangements

$$
\begin{aligned}
{}_nP_r &= n(n-1)\cdots(n-(r-1)) \\
&= n(n-1)\cdots(n-r+1) \\
&= n(n-1)\cdots(n-r+1)\,\frac{(n-r)!}{(n-r)!} \\
&= \frac{n(n-1)\cdots(n-r+1)(n-r)\cdots(3)(2)(1)}{(n-r)!} \\
&= \frac{n!}{(n-r)!}
\end{aligned}
$$

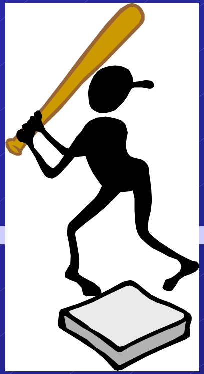
$$\boxed{{}_nP_r = \frac{n!}{(n-r)!}}$$

Permutations: Ordered Arrangements

- Q: How many different batting orders are possible for a baseball team consisting of 9 players?
- A:



Permutations: Ordered Arrangements



- Q: How many different batting orders are possible for the leading **four** batters?
- A:

Permutations: Indistinguishable Objects

- Q: How many different letter arrangements can be formed using the letters **T E N N E S S E E** ?

- A: There are 9! permutations of the letters **T E N N E S S E E** if the letters are <u>distinguishable</u>.

- However, 4 E's are i<u>ndistinguishable</u>. There are 4! ways to order the E's.

Permutations: Indistinguishable Objects, Cont.

- 2 S's and 2 N's are indistinguishable. There are 2! orderings of each.

- Once all letters are ordered, there is only one place for the T.

If the E's, N's, & S's are indistinguishable among themselves, then there are

$$\frac{9!}{(4!\cdot 2!\cdot 2!)} = 3{,}780 \text{ different orderings of}$$

T E N N E S S E E

Permutations: Indistinguishable Objects

Subsets of Indistinguishable Objects

Given n objects of which

a are alike, b are alike, …, and

z are alike

There are $\dfrac{n!}{a!\cdot b!\cdots z!}$ permutations.

Combinations: Unordered Arrangements

- **Combinations:** number of different groups of size *r* that can be chosen from a set of *n* objects (order is irrelevant)

- Q: From a group of 6 people, select 4. How many different possibilities are there?
- A: There are ${}_6P_4$=360 different orderings of 4 people out of 6.

$$6\cdot5\cdot4\cdot3 = 360 = {}_6P_4 = \frac{n!}{(n-r)!}$$

Unordered Example continued

- However the order of the chosen 4 people is irrelevant. There are 24 different orderings of 4 objects.

$$4 \cdot 3 \cdot 2 \cdot 1 = 24 = 4! = r!$$

- Divide the total number of orderings by the number of orderings of the 4 chosen people.

$$\frac{360}{24} = \underline{\text{15 different groups of 4 people}}.$$

Combinations: Unordered Arrangements

The number of ways to choose *r* objects from a group of *n* objects.

C(n,r) or ${}_nC_r$, read as "*n* choose *r*"

$$\boxed{{}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}}$$

Combinations: Unordered Arrangements

- Q: From a group of 20 people, a committee of 3 is to be chosen. How many different committees are possible?
- A:

Combinations: Unordered Arrangements

- Q: From a group of 5 men & 7 women, how many different committees of 2 men & 3 women can be found?
- A:

Teaching Idea

- Advanced web problems on permutations/combinations: http://www.math.uah.edu/stat/comb/index.xml
- The Birthday Problem
  - http://www.mste.uiuc.edu/reese/birthday/intro.html (simulation applet)
  - http://mathforum.org/dr.math/faq.birthdayprob.html (good details)

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Thursday, Feb. 12th, 3:30

- Activity-based Materials for Learning Probability and Statistics
  - Materials reviewed and demonstrated (simulations, discovery learning, group work)
  - Overview of AP statistics

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