10/13/2005 MATH371 Intro. to Prob./Stats. Name: Solution

Dr. Lunsford Midterm Exam (100 Points Total)

Neatly show all work on this test. Clearly indicate your answers. You may use any of the following formulas (if needed) to work this test. If you use one of these formulas, clearly indicate which formula (by its number) you use. Good luck!

(1) $\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^{k}=(a+b)^{n}$ (2) $\sum_{i=1}^{n} i=\frac{n(n+1)}{2}$

(3) $\sum_{n=k}^{\infty} r^{n}=\frac{r^{k}}{1-r},\ |r|<1$ (4) $\sum_{i=1}^{n} i^{2}=\frac{n(n+1)(2n+1)}{6}$

Problem I. Suppose P is a probability function on a sample space S and A and B are events in S such that $P(A)=0.60$, $P(B)=0.40$, and $P(A\cap B)=0.24$. Please answer the following. <u>You must show one intermediate step on each computation for full credit</u>. (4 points each, 24 points total)

a. $P(A\cup B) = P(A)+P(B)-P(A\cap B)$

$= .6+.4-.24 = \boxed{.76}$

b. $P(B\mid A) = \dfrac{P(B\cap A)}{P(A)} = \dfrac{.24}{.60} = \boxed{.40}$

Note: This implies A & B are independent since $P(B\mid A)=P(B)$!

c. $P(A'\cap B) = P(B)-P(A\cap B)$ $\boxed{\text{OR}}$

$= .40-.24 = \boxed{.16}$

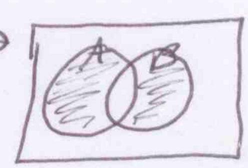
Since A & B are independent then A' & B are independent thus $P(A'\cap B) = P(A')P(B)$

$= .40(.40) = .16$

d. $P(A'\cup B') = P((A\cap B)')$ ← De'morgan's Law

$= 1-P(A\cap B)$

$= 1-.24 = \boxed{.76}$



e. Find the probability that event A or event B but not both occur.

$P(A\cup B) - P(A\cap B)$

$= .76-.24 = \boxed{.52}$

f. Are the events A and B independent events? Why or why not?

Yes (see note on part b above) OR

A & B are independent $\Leftrightarrow$ $P(A\cap B) = P(A)P(B)$

Now $P(A\cap B) = .24$ and $P(A)P(B) = .6(.4) = .24$

Thus A & B are independent.

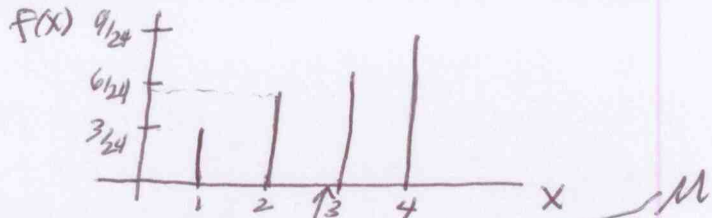
Problem II. Suppose the discrete random variable X has probability mass function $f(x) = \dfrac{2x+1}{24}, x = 1,2,3,4$. Please answer the following, being sure to show all work. (30 points total)

(a) Find μ for this distribution. (10 points)

$$\mu = E[X] = \sum_{x=1}^{4} x f(x) = 1\cdot\frac{3}{24} + 2\cdot\frac{5}{24} + 3\cdot\frac{7}{24} + 4\cdot\frac{9}{24}$$

$$= \frac{70}{24} = 2.91\overline{6}$$

(b) Graph the probability mass function and show μ on your graph. (6 points)



(c) What is $P(X \geq 1.5)$? (4 points)

$$P(X \geq 1.5) = P(X=1) + P(X=2) + P(X=3) = \frac{21}{24}$$

$$\boxed{\text{OR}}\; P(X \geq 1.5) = 1 - P(X < 1.5) = 1 - P(X=1) = \frac{21}{24}$$

(d) Suppose you can generate pseudo random numbers via a uniform random number generator on your computer. Use the notation $Y = unif(0,1)$ to indicate a uniform random number from 0 to 1. Use pseudo code to write an algorithm that will *print out 10 values of the random variable* X (where X is distributed as specified above) *by using the uniform (from 0 to 1) random number generator.* (10 points)

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For i = 1, ..., 10
    y = unif(0,1)
    IF y ≤ .125 then          → 3/24
        X = 1
    ELSE IF y ≤ .333 then     → 8/24
        X = 2
    ELSE IF y ≤ .625 then     → 15/24
        X = 3
    ELSE
        X = 4
    ENDIF
    PRINT X
END For
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Problem III. We are going to play the following game: We will take turns rolling a fair six-sided die. I will roll first, you will roll next, then me, etc. We will stop the game when the first 1 or 6 appears on a roll. I will win if that roll is my roll, otherwise you will win. Please answer the following questions. (16 points total)

(a) What is the probability that you will win on *your first roll*? On *your second roll*? On *your third roll*? Clearly indicate your answers and please leave them in combinatorial form. (8 points)

Dr. L. lose — you win

Win on 1st roll : $P(LW) = \frac{2}{3} \cdot \frac{1}{3}$

Dr. L. — you

Win on 2nd roll : $P(LLLW) = \left(\frac{2}{3}\right)^3 \cdot \frac{1}{3}$

Wind on 3rd roll : $P(LLLLLW) = \left(\frac{2}{3}\right)^5 \cdot \frac{1}{3}$

(b) Find the probability that you will win the game. (8 points)

$$P(\text{Win}) = P(LW) + P(LLLW) + P(LLLLLW) + \cdots$$

$$= \sum_{i=1}^{\infty} \left(\frac{2}{3}\right)^{2i-1} \left(\frac{1}{3}\right) = \frac{1}{2} \sum_{i=1}^{\infty} \left(\frac{4}{9}\right)^i$$

→ Now use formula (3)

$$= \frac{1}{2} \cdot \frac{\frac{4}{9}}{1 - \frac{4}{9}} = \boxed{\frac{2}{5}}$$

Problem IV. A mortgage company had applications for a home mortgage loan from 25 equally financially qualified applicants. Seven of these applications were from minority applicants. Because of limited resources, the mortgage company was only able to approve ten of the applications. None of the minority applicants were approved. (10 points total)

(a) If the company was randomly choosing from the equally qualified applicants, what is the probability that at least one of the approved applications would have been from a minority applicant? (8 points)

P(at least one min. approve) = 1 − P(no min. approve)

$$= 1 - \frac{\binom{7}{0}\binom{18}{10}}{\binom{25}{10}} = 1 - .0134 = .9866$$

(b) This particular mortgage company has been accused of discriminating against minority applicants. Do you think there is any evidence to support this accusation? Why or why not? (2 points)

Yes! There is a 98.7% chance that at least one of the minority applicants would have been chosen if choosing randomly. Thus it would be rare (but not impossible), if choosing from equally qualified applicants, for a nonbiased company to get these results.

$P(C|A_3)=.07$ $P(C|A_1)=.47$ $P(C|A_2)=.21$

Problem V. Online chat rooms are dominated by the young. If we look only at adult Internet users (age 18 and over), 47% of the 18 to 29 age group chat, as do 21% of those aged 30 to 49 and just 7% of those 50 and over. Suppose that 29% of adult internet users are age 18 to 29, another 47% are 30 to 49 years old, and the remaining 24% are age 50 or older. Let C be the event that an adult internet user chats, A_1 be the event that an adult internet user is age 18 to 29, A_2 be the event that an adult internet user is age 30 to 49, and A_3 be the event an adult internet user is age 50 or older. What percent of the adult internet users who chat are in the 18 to 29 age group? Please be sure to write all probabilities you use for this computation in terms of the event names given above. (10 points total)

$P(A_1)=.29$

$P(A_3)=.24$

$P(A_2)=.47$

$$P(A_1|C)=\frac{P(A_1\cap C)}{P(C)}=\frac{P(C|A_1)P(A_1)}{P(C)}$$

$$=\frac{P(C|A_1)P(A_1)}{P(C|A_1)P(A_1)+P(C|A_2)P(A_2)+P(C|A_3)P(A_3)}$$

$$=\frac{.47(.29)}{.47(.29)+(.21)(.47)+(.07)(.24)}=.5413$$

$\boxed{\text{54.13\% who chat are in the 18 to 29 age group.}}$

Problem VI. On a separate piece of paper, work ONE of the following problems. Clearly indicate which problem you want me to grade. BONUS: Work another of these problems (in this case clearly indicate which problem counts towards the test and which problem is your bonus problem). (10 points)

I specifically mentioned (in class) both of these problems as excellent problems for the midterm...

(a) If A and B be are independent events in a sample space S then show that A' and B' are also independent. See Th. 2.4-1 on page 97 and your notes — we proved this in class I believe...

(b) If X is a discrete random variable with the discrete uniform distribution, i.e. the p.m.f. for X is $f(x)=\frac{1}{m}, x=1,2,\ldots,m$, then show that $\sigma^2=\frac{m^2-1}{12}$. You may use the fact that $\mu=\frac{m+1}{2}$. See Example 3.2-8 on page 125, you should be able to fill in the algebraic details...

(c) If Y is a random variable such that $E[2Y+3]=6$ and $E\left[(2Y+3)^2\right]=100$ then find the mean of Y, μ_Y, and the standard deviation of Y, σ_Y.

Problem VI (c)

Let $X = 2Y + 3$ then

(1) $\mu_X = 2\mu_Y + 3$ and

(2) $\sigma_X = 2\sigma_Y$

Now $\mu_X = E[X] = E[2Y+3] = 6$

thus by (1) we have: $6 = 2\mu_Y + 3$

$\Rightarrow \boxed{\mu_Y = \frac{3}{2}}$

Also $\sigma_X^2 = E[X^2] - (E[X])^2$

$= E\{(2Y+3)^2] - 6^2$

$= 100 - 36 = 64 \Rightarrow \sigma_X = 8$

Thus by (2) we have

$\sigma_X = 2\sigma_Y \Rightarrow 8 = 2\sigma_Y \Rightarrow \boxed{\sigma_Y = 4}$