

I. An urn contains 10 yellow chips and 6 blue chips. Chips are drawn from the urn without replacement. What is the probability that the third blue chip is drawn on the 5<sup>th</sup> draw? You may leave your answer in combinatorial form. (7 points)

A = Event 2 Blues on 1<sup>st</sup> 4 draws

B = Event Blue on 5<sup>th</sup> draw

$$P(A \cap B) = P(B|A)P(A) = \frac{4}{12} \cdot \frac{\binom{6}{2} \binom{10}{2}}{\binom{16}{4}}$$

II. The probability that a woman hits a target is 5/7. She starts to shoot until she hits the target for the first time. Let the random variable  $X$  be the shot number on which she first hits the target. What is  $S_X$ , the set of all possible values of  $X$ ? (2 points)

$$S_X = \{1, 2, 3, \dots\}$$

III. Consider the random variable defined in Problem II above. Find the following probabilities (you may leave your answers in combinatorial form). (10 points total)

(a)  $P(X = 5)$  (4 points) =  $\frac{2}{7} \cdot \frac{2}{7} \cdot \frac{2}{7} \cdot \frac{2}{7} \cdot \frac{5}{7} = \left(\frac{2}{7}\right)^4 \left(\frac{5}{7}\right)$

(b)  $P(X = k)$  where  $k \in S_X$  (4 points)  $\left(\frac{2}{7}\right)^{k-1} \left(\frac{5}{7}\right)$

(c) What assumption are you making concerning the woman's shots when you compute the above probabilities? (2 points)

The shots are independent.

IV. Six balls are drawn, *with replacement*, from an urn with 10 red and 11 blue balls. An example outcome is BRBBRR. Find the following probabilities. You may leave your answers in combinatorial form. (9 points total)

(a)  $P(\text{RBBRRB})$  (4 points)  $\frac{10}{21} \cdot \frac{11}{21} \cdot \frac{11}{21} \cdot \frac{10}{21} \cdot \frac{10}{21} \cdot \frac{11}{21} = \left(\frac{10}{21}\right)^3 \left(\frac{11}{21}\right)^3$

(b) Exactly three red balls are drawn. (5 points)

→ count all outcomes with 3 R balls:  $\binom{6}{3} \left(\frac{10}{21}\right)^3 \left(\frac{11}{21}\right)^3$

V. In a certain population of voters, 40% identify as Republican, 35% identify as Democratic, and 25% identify as Independent. Suppose 20% of Republicans approve of the job Barack Obama is doing as President of the United States, 60% of Democrats approve, and 30% of Independents approve. Please answer the following being sure to clearly identify all event names used (i.e. let A be the event that...). (12 points total)

(a) Based on the above information, what percent of the voters in this population approve of the job Barack Obama is doing as President of the United States? (6 points)

Let R be event voter identifies as Republican  
 D " " " " " Democratic  
 I " " " " " Independent  
 A " " " approves.

$$\begin{aligned}
 P(A) &= P(A|D)P(D) + P(A|R)P(R) + P(A|I)P(I) \\
 &= (.60)(.35) + (.20)(.40) + (.30)(.25) \\
 &= 0.365 \quad \boxed{36.5\%}
 \end{aligned}$$

(b) Suppose a voter from this population is randomly chosen and they do not approve of the job Barack Obama is doing as President of the United States. What is the probability that this voter is an Independent? (6 points)

$$\begin{aligned}
 P(I|A') &= \frac{P(I \cap A')}{P(A')} \\
 &= \frac{P(A'|I)P(I)}{1 - P(A)} = \frac{.70(.25)}{0.635}
 \end{aligned}$$

↑  
computed above

$$\boxed{= .2756}$$