10/12/2004 MATH371 Intro. to Prob./Stats. Name: Solution

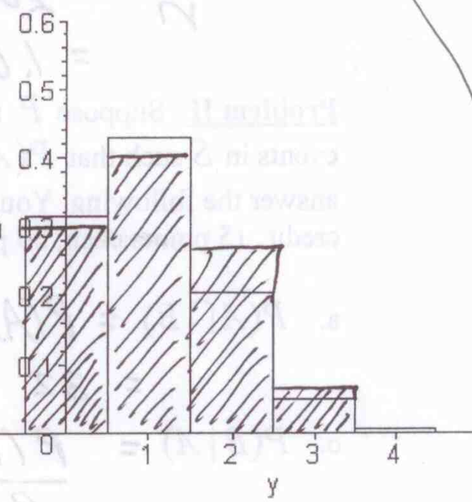
Dr. Lunsford Test 1 (100 Points Total)

Neatly show all work on this test. Clearly indicate your answers. Unless otherwise indicated, you may leave your answers in combinatorial form. Good luck!

Do count order: $25 + 25\cdot 24 + 25\cdot 24\cdot 23 + 25\cdot 24\cdot 23\cdot 22$

Problem I. A trick-or-treater at Dr. L.'s house must first roll a fair four-sided die before getting their treat(s). Dr. L. will then draw (without replacement from an urn of candy) the number of candies as specified on the die to give to the trick-or-treater. Suppose the candy urn contains 10 Snickers bars and 15 Reeses Peanut Butter cups. Let the random variable X denote the face value on the die and the random variable Y denote the number of Snicker bars drawn. Below you are given a graph of the probability mass function (p.m.f.) for Y and a table that contains the values of the p.m.f. for Y. Please answer the following questions. (28 points total)

y	0	1	2	3	4
$f(y) = P(Y = y)$	0.3139	0.4291	0.2042	0.0486	0.0042



(a) How many outcomes are possible for this random experiment? An example of an outcome is a die roll of 4 and a draw of 3 Snicker bars and 1 Reeses cup. (4 points)

$\binom{25}{1} + \binom{25}{2} + \binom{25}{3} + \binom{25}{4}$

$\uparrow$ die is 1 $\quad \uparrow$ die is 2 $\quad$ (Do not count order)

(b) If the die roll is a 3, what is the probability that the trick-or-treater will get 2 Snicker bars? Be sure to write all probabilities you use/compute in terms of the random variables X and Y. (6 points)

$$P(Y=2 \mid X=3) = \frac{\binom{10}{2}\binom{15}{1}}{\binom{25}{3}}$$

(c) What is the probability the trick-or-treater will receive two Snicker bars? You must show all probabilities you use/compute in terms of the random variables X and Y. Note: The answer is on this test paper – you must show how the answer is reached! (6 points)

$$P(Y=2) = P(Y=2 \mid X=2)P(X=2) + P(Y=2 \mid X=3)P(X=3) + P(Y=2 \mid X=4)P(X=4) = \frac{\binom{10}{2}}{\binom{25}{2}}\cdot\frac{1}{4} + \frac{\binom{10}{2}\binom{15}{1}}{\binom{25}{3}}\cdot\frac{1}{4} + \frac{\binom{10}{2}\binom{15}{2}}{\binom{25}{4}}\cdot\frac{1}{4} =$$

(d) Find the mean of the distribution for the random variable Y. Show the formula you use and how you are plugging the data into the formula to compute this value. (4 points)

$$\mu = \sum y f(y) = 0(.3139) + 1(.4291) + 2(.2042) + 3(.0486) + 4(.0042) = 1.0001$$

Problem I, continued.

(e) Suppose we run this experiment 20 independent times (i.e. we will always make sure that the candy urn contains 10 Snickers bars and 15 Reeses Peanut Butter cups) and observe the following values of Y: 1, 0, 1, 2, 0, 0, 2, 2, 0, 2, 1, 2, 1, 1, 0, 3, 1, 0, 1, 1
Find the relative frequencies for this data and plot the corresponding histogram on the same axes as the p.m.f. of Y. Please shade your histogram rectangles so that I can distinguish your histogram from the p.m.f. of Y. (4 points)

Y	0	1	2	3
freq	6	8	5	1
Rel freq	6/20	8/20	5/20	1/20

See shaded histogram on first page...

(f) Find the mean of the data in part (e). Show the formula you use and how you are plugging the data into the formula to compute this value. (4 points)

$$\bar{X} = \frac{\Sigma X}{n} = \frac{1}{20}\left[0(6) + 1(8) + 2(5) + 3(2)\right]$$
$$= 1.05$$

Problem II. Suppose P is a probability function on a sample space S and A and B are events in S such that $P(A) = 0.52$, $P(B) = 0.50$, and $P(A \cup B) = 0.70$. Please answer the following. You must show one intermediate step on each computation for full credit. (5 points each, 30 points total)

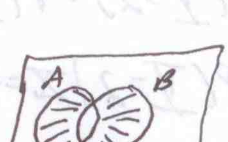
a. $P(A \cap B) = P(A) + P(B) - P(A \cup B) = .52 + .50 - .70$
$= .32$

b. $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{.32}{.52} = .6154$

c. $P(A^C \cap B) = P(B) - P(A \cap B)$
$= .50 - .32 = .18$

d. $P(A^C \cap B^C)$
$= P((A \cup B)^c) = 1 - P(A \cup B) = 1 - .70 = .30$

e. Find the probability that event A or event B but not both occur.



$P(A \cup B) - P(A \cap B) = .70 - .32 = .38$

f. Are the events A and B independent events? Why or why not?

$P(A \cap B) = .32$
$P(A)P(B) = (.52)(.50) = .26$

Since $P(A \cap B) \neq P(A)P(B)$ then A & B are not independent.

$$P(\text{red}) = \frac{6}{15}, \quad P(\text{white}) = \frac{9}{15}$$

Problem III. An urn contains 6 red and 9 white balls. Six balls are drawn from the urn with replacement. An example outcome is the draw RRWWWW. Please answer the following: (20 points total)

(a) Find the probability of getting the outcome RRWWWW. (4 points)

$$\left(\frac{6}{15}\right)^2\left(\frac{9}{15}\right)^4$$

(b) Are the outcomes for this random experiment equally likely? Why or why not? (4 points)

Because it is more likely to draw a W than a R, the outcome RRRRRR is less likely to occur than the outcome WWWWWW. ∴ the outcomes are not equally likely.

(c) What is the probability that at least one red ball will be drawn? (6 points)

$$P(\text{at least one red}) = 1 - P(\text{no red})$$

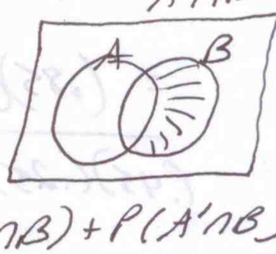
$$= 1 - P(\text{all white}) = 1 - \left(\frac{9}{15}\right)^6$$

(d) Find the probability that the 6th ball drawn is the 4th white ball drawn? (6 points)

$$\underbrace{\_\ \_\ \_\ \_\ \_}_{3\ Ws}\ W \qquad \binom{5}{3}\left(\frac{6}{15}\right)^2\left(\frac{9}{15}\right)^3\cdot\left(\frac{9}{15}\right)$$

Problem IV. Let A and B be events in a sample space S with $P(B) \neq 0$. Show that $P(A^C \mid B) = 1 - P(A \mid B)$. (6 points)

$A' \cap B$



$$1 - P(A|B) = 1 - \frac{P(A \cap B)}{P(B)} \quad (\text{Defn. of } P(A|B))$$

$$= \frac{P(B) - P(A \cap B)}{P(B)} \quad (\text{Algebra})$$

$$= \frac{P(A' \cap B)}{P(B)} \quad (\text{since } P(B) = P(A \cap B) + P(A' \cap B))$$

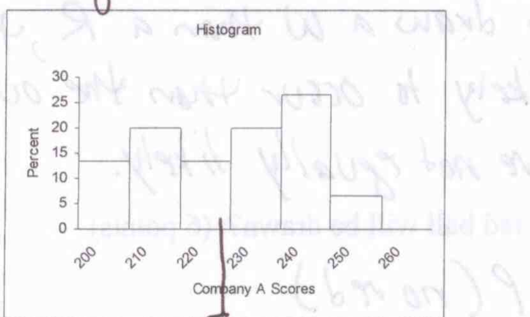
$$= P(A'|B) \quad (\text{definition of } P(A'|B))$$

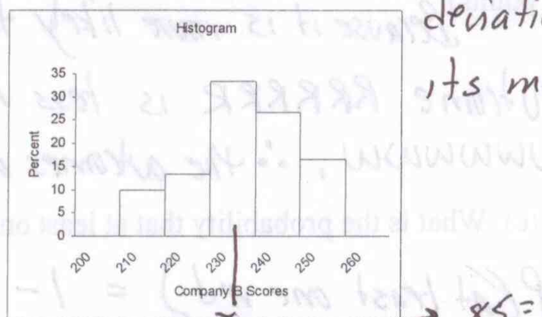
//

i.e. higher than

Problem V. Below you are given the histograms associated with two data sets. The data sets are the test scores on a standardized aptitude test given to applicants who have applied to Company A and to applicants who have applied to Company B. The sample mean for Company A's scores is 228.73 and the sample mean for Company B's scores is 236.23. Without doing any computations, determine which company has the smallest variance for the scores. Justify your answer. (4 points)

Company B's data has the smaller variance. This is because more of Company B's data is closer to $\bar{x} = 236.23$ than compared to the amount of Company A's data to 228.73. (i.e. B's data has less deviation from its mean).





$\bar{x}$ $\bar{x}$

Problem VI. Suppose that in the coming election, 85% of Republicans vote for Bush, 10% of Democrats vote for Bush, 45% of Independents vote for Bush, and the voters consist of 35% Republicans, 40% Democrats, and 25% Independents. Suppose one of these voters is chosen at random. Let the R be the event the voter is a Republican, D be the event the voter is a Democrat, I be the event the voter is an Independent, and B be the event the voter votes for Bush. (12 points total)

$.85 = P(B|R)$, $.10 = P(B|D)$, $.45 = P(B|I)$, $P(R) = .35$, $P(D) = .40$, $P(I) = .25$

(a) If the randomly selected voter votes for Bush, then what is the probability that the voter is a Republican? To receive full credit you must write all probabilities that you use for this computation in terms of the event names given above. (10 points)

$$P(R|B) = \frac{P(R \cap B)}{P(B)} = \frac{P(B|R)P(R)}{P(B|I)P(I) + P(B|D)P(D) + P(B|R)P(R)}$$

Law of Total Prob

$$= \frac{(.85)(.35)}{(.45)(.25) + (.10)(.40) + (.85)(.35)} = \boxed{.661}$$

(b) Compare the probability computed in part (a) to the probability that a randomly selected voter is a Republican. Explain why the numerical difference in these two probabilities makes sense. (2 points)

$P(R|B) = .66 > .35 = P(R)$. Knowing that a voter voted for Bush makes it more likely that the voter is a Republican. This makes sense b/c while there are essentially equal #'s of R's, D's & I's in the population (about 1/3 each) the R voted overwhelming for B & will thus make up a larger % of B voters! (compared to the D's & I's)