

Pledge:

11/14/2005  
Dr. Lunsford

MA371 Intro. To Prob. & Stats.  
Quiz 6

Name: Solution  
(20 Points Total)

You must show all work on this quiz for full credit. Good luck!

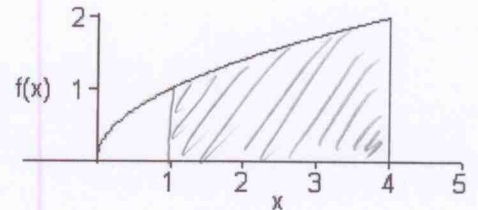
I. Suppose the random variable  $X$  has p.d.f. given by  $f(x) = \frac{3}{16}\sqrt{x}$  for  $0 \leq x \leq 4$ . Below you are

given a graph of this p.d.f. Please answer the following:  
(12 points total)

p.d.f. for  $X$

(a) Find the mean of the random variable  $X$ . (4 points)

$$E[X] = \int_{\mathbb{R}} x f(x) dx = \int_0^4 x \cdot \frac{3}{16} \sqrt{x} dx$$
$$= \frac{12}{5}$$



(b) Find the probability that  $X$  is at least 1. Clearly represent this probability graphically above. (4 points)

$$P(X \geq 1) = \int_1^4 \frac{3}{16} \sqrt{x} dx = \frac{7}{8}$$

OR

$$= 1 - P(X \leq 1) = 1 - \int_0^1 \frac{3}{16} \sqrt{x} dx = \frac{7}{8}$$

(c) Let  $W = X^2$ . Find the probability density function for  $W$ . Be sure to indicate where  $W$  is defined. (4 points)

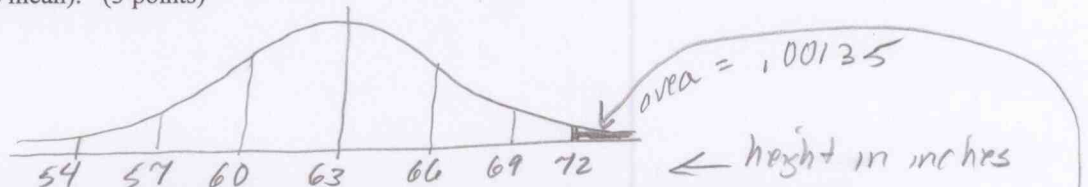
$$0 \leq x \leq 4 \Rightarrow 0 \leq w \leq 16$$

$$G(w) = P(W \leq w) = P(X^2 \leq w) = P(X \leq \sqrt{w}) = F(\sqrt{w})$$

where  $F'(x) = \frac{3}{16}\sqrt{x}$ . Thus  $g(w) = G'(w) = F'(\sqrt{w}) \cdot \frac{1}{2\sqrt{w}} = \frac{3}{32} w^{-1/4}$

II. Suppose the heights of American women are approximately normally distributed with mean 63 inches and standard deviation 3 inches. Please answer the questions below: (8 points total)

(a) Draw a graph of the p.d.f. for this distribution below. You should clearly indicate the values of points of interest in the domain (i.e. the mean and values within plus or minus one, two, and three standard deviations from the mean). (3 points)



(b) How likely is it that an American woman selected at random will be over 6 feet tall? Show all calculator input for this computation. Clearly represent this likelihood on your graph above. (3 points)

$$P(X \geq 72) = \text{normalcdf}(72, 1E99, 63, 3)$$
$$= .00135$$

(c) Suppose that ten American women are selected at random. Let the random variable  $Y$  be the number of women in this sample who are over 6 feet tall. How is the random variable  $Y$  distributed (you must give the name of the distribution and the values of any relevant parameters)? (2 points)

$Y$  is binomial w/  $n=10$  &  $p=.00135$