

Pledge:

10/31/2005
Dr. Lunsford

MA371 Intro. To Prob. & Stats.
Quiz 5

Name: Solution
(20 Points Total)

You must show all work on this quiz for full credit. You may use your calculator and the front flap of your textbook to work this quiz. Clearly indicate all formulas you use and all calculator input. Good luck!

I. According to a recent Pew poll (October 12-24, 2005), 40% of Americans approve of the job George W. Bush is doing as President. Use this information to answer the following questions: (15 points total)

(a) Suppose we start to randomly select Americans and ask them if they approve of the job George W. Bush is doing as President (i.e. we will get a random sample). On average, how many Americans will we need to select before we find an American who approves? (2 points)

$$\text{Geometric } \mu = \frac{1}{p} = \frac{1}{.4} = 2.5$$

(b) How likely is it that the fourth American we select is the first American in our sample who approves? (3 points)

$$P(X=4) = f(4) = (.6)^3(.4) = .0864 \quad \boxed{\text{An } 8.64\% \text{ chance}}$$

(c) Suppose we eventually sample 200 Americans. Let X be the number in our sample who approve of the job George W. Bush is doing as President. How is the random variable X distributed (you must give the name of the distribution and the values of any relevant parameters)? (3 points)

X is binomial with $n=200$ and $p=.4$

(d) On average, how many Americans in our sample in part (c) above can we expect to approve? (2 points)

$$\mu = np = 200(.4) = 80$$

(e) Suppose in our sample in part (c) above we get 98 or more Americans who approve. Would you be surprised? Why or why not? You must back up your answer with an appropriate probability computation. (5 points)

$$\begin{aligned} P(X \geq 98) &= 1 - P(X \leq 97) \\ &= 1 - \text{binomcdf}(200, .4, 97) \\ &= .0061 \end{aligned}$$

I would be very surprised since this is an unlikely event.

Note: The probability that X equals any value from 0 to 200 is going to

II. Show that if X has a geometric distribution with parameter p , then the mean of X is $1/p$. Clearly indicate your method and show all of your work. (5 points)

$$E[X] = \sum_{x \in S_X} x f(x) = \sum_{x=1}^{\infty} x (1-p)^{x-1} p \quad (\text{double but involves a slick trick...})$$

so use m.g.f instead!

$$m(t) = pe^t (1 - (1-p)e^t)^{-1}$$

$$m'(t) = pe^t (1 - (1-p)e^t)^{-1} + pe^t (-1) (1 - (1-p)e^t)^{-2} (-1-p)$$

$$m'(0) = p(p)^{-1} + p(-1)(p)^{-2}(-1+p)$$

$$= 1 - \frac{p}{p^2}(-1+p) = \boxed{\frac{1}{p}}$$

BONUS: What was the best Halloween costume you ever wore? (2 points)

Bugs Bunny