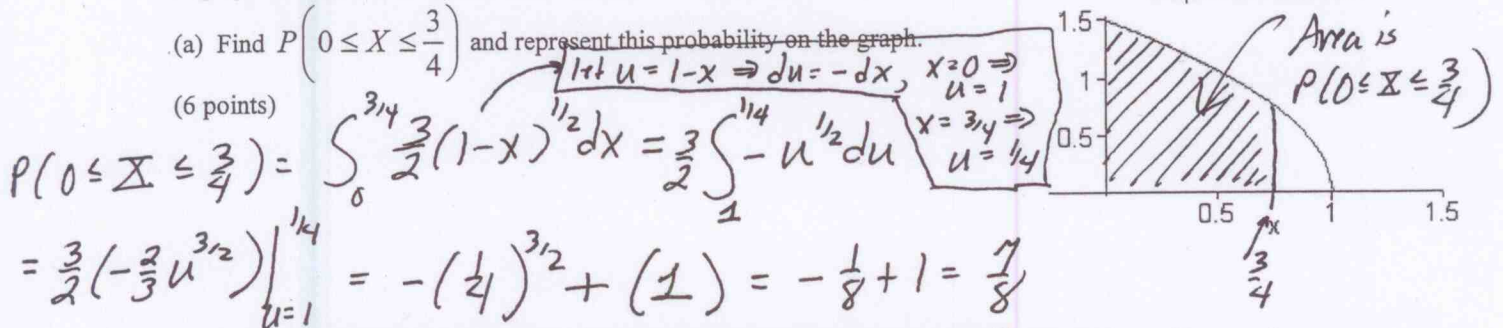


I. Suppose the random variable  $X$  has the probability density function  $f(x) = \frac{3}{2}\sqrt{1-x}$  for  $0 \leq x \leq 1$ .

A graph of the p.d.f. is given below. Please answer the following:

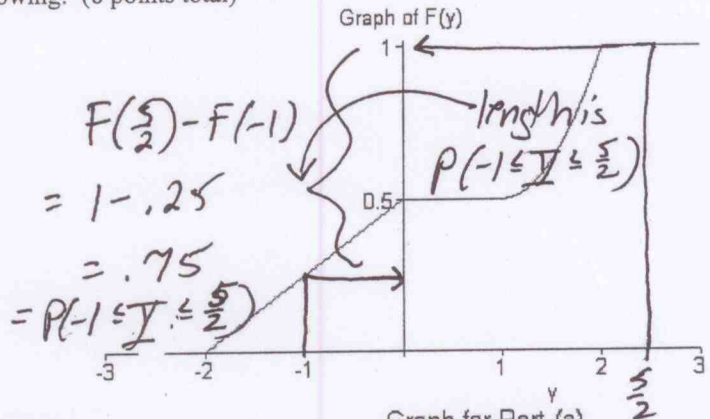
(a) Find  $P\left(0 \leq X \leq \frac{3}{4}\right)$  and represent this probability on the graph.

(6 points)



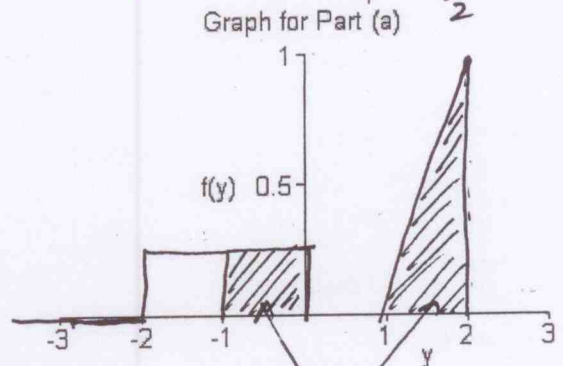
II. Suppose the random variable  $Y$  has the cumulative distribution function  $F$  given below. You are also given a graph of the c.d.f. Please answer the following: (8 points total)

$$F(y) = \begin{cases} 0, & y \leq -2 \\ \frac{1}{4}y + \frac{1}{2}, & -2 \leq y \leq 0 \\ \frac{1}{2}, & 0 \leq y \leq 1 \\ \frac{1}{2}y^2 - y + 1, & 1 \leq y \leq 2 \\ 1, & y \geq 2 \end{cases}$$



(a) Find the probability density function,  $f(y)$ , for  $Y$  and graph it on the axes provided below. (5 points)

$$f(y) = \begin{cases} \frac{1}{4}, & -2 < y < 0 \\ y-1, & 1 < y < 2 \\ 0, & \text{elsewhere} \end{cases}$$



(b) Graphically represent (but DO NOT FIND!) the value of  $P\left(-1 \leq Y \leq \frac{5}{2}\right)$  on the graphs of both the c.d.f. and the p.d.f. for  $Y$ . (3 points)

$$F\left(\frac{5}{2}\right) - F(-1) = 1 - \frac{1}{4} = \frac{3}{4}$$

III. A discrete random variable, say  $X$ , has the moment generating function

$M(t) = \frac{1}{4} + \frac{3}{8}e^t + \frac{1}{8}e^{2t} + \frac{1}{4}e^{3t}$ . Find  $P(X=2)$  and  $E[X]$ . Clearly indicate your answers. (6 points)

$$m(t) = \sum f(x)e^{tx} \text{ so in this case } P(X=2) = \frac{1}{8}$$

$$E[X] = m'(0) = \frac{3}{8} + \frac{1}{4} + \frac{3}{4} = \frac{11}{8}$$

$$m'(t) = \frac{3}{8}e^t + \frac{1}{4}e^{2t} + \frac{3}{4}e^{3t}$$