MATH 371 - Quiz ~~3~~ 4 - Solution

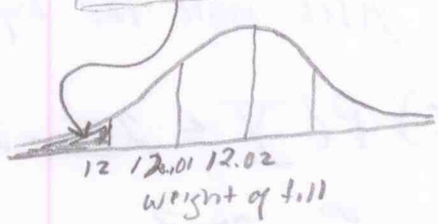
I. $F(y) = P(Y \le y) = P(-2\theta \ln(X) \le y) = P\left(\ln X \ge \frac{-y}{2\theta}\right)$

$= P(X \ge e^{-y/(2\theta)}) = \int_{e^{-y/(2\theta)}}^{1} \theta x^{\theta-1}\,dx = x^{\theta}\Big|_{e^{-y/(2\theta)}}^{1}$

$= 1 - (e^{-y/(2\theta)})^{\theta} = 1 - e^{-y/2}, \quad 0 \le y < \infty$

Thus
$f(y) = F'(y) = \frac{1}{2} e^{-y/2}, \quad 0 \le y < \infty$

II (a) normalcdf$(-1E99, 12, 12.02, .1) = \boxed{.02275}$



(b) Let Y be the # of cans chosen that weigh 12 oz. or less. Then $Y \sim b(n=50, p=.02275)$. Thus $P(Y \ge 1) = 1 - P(Y=0)$

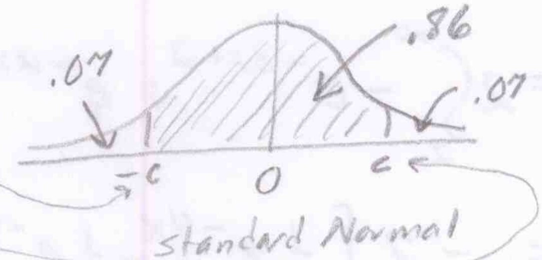
$= 1 - (.97725)^{50} = \boxed{.6836}$

III $P(|Z| < c) = .86 \iff P(-c < Z < c) = .86$

So invNorm$(.07) = -1.476$

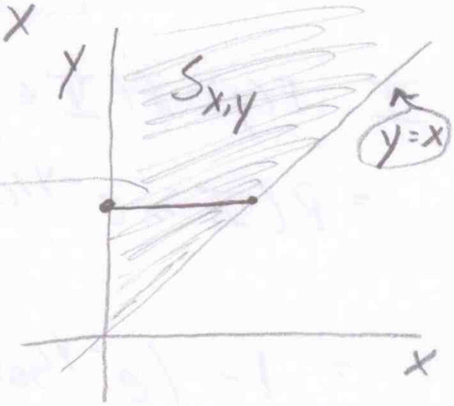
OR invNorm$(.93) = 1.476$

$\therefore \boxed{c = 1.476}$



IV

(a) $f_Y(y) = \int_{\text{all } x} f(x,y)\,dx = \int_0^y 2e^{-x-y}\,dx$



$= 2e^{-y}\int_0^y e^{-x}\,dx$

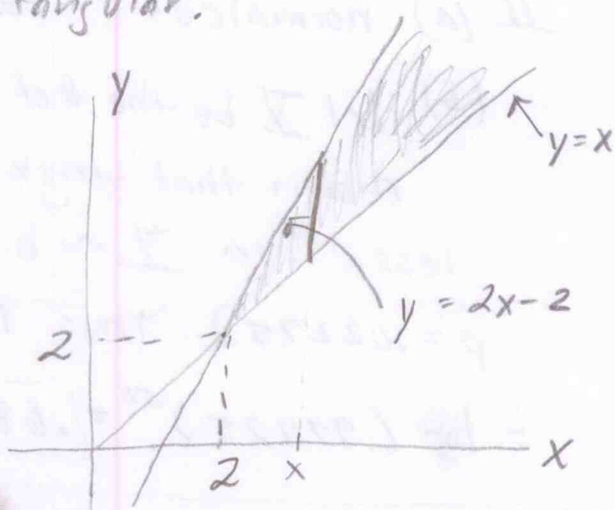
$= 2e^{-y}\left[1-e^{-y}\right]$

(b) $2e^{-y}\left[1-e^{-y}\right]\left(2e^{-2x}\right) \neq 2e^{-x-y}$

$\forall\ (x,y) \in S_{x,y}$. $\therefore$ X and Y are not independent.

Also note the support is not rectangular.

(c) $P(Y \le 2X-2)$



$$= \int_2^{\infty}\int_x^{2x-2} 2e^{-x-y}\,dy\,dx$$

$$= \int_2^{\infty} 2e^{-x}\left[-e^{-2x+2} + e^{-x}\right]dx$$

$$= 2\int_2^{\infty} -e^{-3x+2} + e^{-2x}\,dx = 2\left[\tfrac{1}{3}e^{-3x+2} - \tfrac{1}{2}e^{-2x}\right]_2^{\infty}$$

$$= -2\left[\tfrac{1}{3}e^{-4x} - \tfrac{1}{2}e^{-4x}\right] = \tfrac{1}{3}e^{-4x}$$