

I. A cookie jar contains 6 chocolate chip and 5 peanut butter cookies. The keeper of the cookie jar has a fair four-sided die. The keeper rolls the die and then you are allowed to draw out the number of cookies as specified on the die (i.e. if the die face is a 3 you will draw out three cookies (without replacement since you will also be eating the cookies!)). Let $X = n$ denote the value of the die roll where $n = 1, 2, 3, 4$ and $Y = k$ denote the number of chocolate chip cookies drawn from the jar, where $k = 0, \dots, 4$. Please find the indicated probabilities. Be sure to write all probabilities you compute using the event names above.

(a) What is the probability that you will get 3 chocolate chip cookies given that the die roll was a 4? (3 points)

$$P(Y=3|X=4) = \frac{\binom{6}{3}\binom{5}{1}}{\binom{11}{4}} = \frac{6 \cdot 5 \cdot 4 \cdot 5}{11 \cdot 10 \cdot 9 \cdot 8} = \frac{5 \cdot 4 \cdot 5 \cdot 3}{2 \cdot 3} = \frac{10}{33}$$

$\frac{(20)(5)}{330} = \frac{10}{33}$ ✓

(b) What is the probability that you will get 3 chocolate chip cookies? (4 points)

$$P(Y=3) = P(Y=3|X=1)P(X=1) + P(Y=3|X=2)P(X=2) + P(Y=3|X=3)P(X=3) + P(Y=3|X=4)P(X=4)$$

$$= \frac{\binom{6}{3}\binom{5}{0}}{\binom{11}{3}} \cdot \frac{1}{4} + \frac{\binom{6}{3}\binom{5}{1}}{\binom{11}{4}} \cdot \frac{1}{4} = \frac{20}{105} \cdot \frac{1}{4} + \frac{10}{33} \cdot \frac{1}{4} = \frac{1}{4} \left[\frac{4}{33} + \frac{10}{33} \right] = \frac{1}{4} \cdot \frac{14}{33} = \frac{7}{66}$$

(c) Given that you drew 3 chocolate chip cookies, what is the probability that the die roll was a 4? (4 points)

$$P(X=4|Y=3) = \frac{P(Y=3|X=4)P(X=4)}{P(Y=3)}$$

$$= \frac{\left[\frac{\binom{6}{3}\binom{5}{1}}{\binom{11}{4}} \right] \cdot \frac{1}{4}}{\left[\frac{\binom{6}{3}\binom{5}{0}}{\binom{11}{3}} \right] \cdot \frac{1}{4} + \left[\frac{\binom{6}{3}\binom{5}{1}}{\binom{11}{4}} \right] \cdot \frac{1}{4}} = \frac{\frac{10}{33} \cdot \frac{1}{4}}{\frac{2}{66}} = \frac{10 \cdot 66}{7 \cdot 33} \cdot \frac{1}{4} = \frac{20}{28} = \frac{10}{14} = \frac{5}{7} = .714$$

(d) Explain why the answer in part (c) makes sense. (2 points)

Know that we got 3 choc chips makes it much more likely that we rolled a four on the die (not: It is impossible to have rolled a 1 or 2!)

II. An urn contains 10 blue and 8 red balls. Pablo draws out two balls with replacement. Then Claudia also draws out two balls with replacement.

(a) What is the probability that Pablo will draw two blue balls and Claudia will draw two red balls? (3 points)

Prob Pablo gets 2 blue $\frac{10}{18} \cdot \frac{10}{18} \cdot \frac{8}{18} \cdot \frac{8}{18} = \left(\frac{5}{9}\right)^2 \left(\frac{4}{9}\right)^2 = \frac{(25)(16)}{9^4} = .0610$

Prob Claudia get 2 red

(b) What is the probability that Pablo draws two balls of the same color, Claudia draws two balls of the same color, and that Pablo's and Claudia's colors match? (4 points)

$$P(\text{Pablo 2 blue and Claudia 2 blue}) \text{ or } P(\text{Pablo 2 red and Claudia 2 red})$$

$$= \left(\frac{10}{18}\right)^4 + \left(\frac{8}{18}\right)^4 = \left(\frac{5}{9}\right)^4 + \left(\frac{4}{9}\right)^4 = .1343$$