

Please sign your name for the honor pledge:

9/25/2007
Dr. Lunsford

MA371 Intro. To Prob. & Stats.
Quiz 3

Name: Solution
(25 Points Total)

I. Dr. L. has twelve cans of cat food for Kitteny Thang (KT) and Hep Kat. The twelve cans, 5 tuna flavor and 7 liver flavor, are stored in a bin on a top shelf. Each day Dr. L. reaches into the bin, randomly selects two cans of cat food and splits the contents for the cats (i.e. each cat gets $\frac{1}{2}$ of each can). Please answer the following. Please show your answers in both combinatorial and decimal form. (3 points each - 6 points total)

(a) KT loves tuna! Is there a better than 50% chance that she will get some tuna on the first feeding day with these 12 cans? I.e. find the probability that KT will get some tuna on the first day Dr. L. selects cat food.

$$1 - P(\text{no tuna}) = 1 - \frac{\binom{7}{2}}{\binom{12}{2}} = 1 - \frac{7 \cdot 6}{12 \cdot 11} = \frac{15}{22} = .6818$$

Yes! There is a 68% chance KT will get some tuna.

(b) What is the probability that the 8th can of cat food chosen will be the 3rd can of tuna chosen (i.e. the second can chosen on the 4th day will be tuna)?

$$\frac{\binom{5}{2} \binom{7}{5}}{\binom{12}{7}} \cdot \frac{3}{5} = P(B|A)P(A) = \frac{35}{220} = .1591$$

Annotations:
 - $\binom{5}{2}$: 2 cans tuna
 - $\binom{7}{5}$: 5 cans liver
 - $\frac{3}{5}$: 3 cans tuna left, P(B|A)
 - $\binom{12}{7}$: 1st 7 cans chosen

$P(\text{2 cans tuna in 1st 7 chosen AND tuna on 8th})$

II. The discrete random variable X has probability mass function $f(x) = \frac{x}{55}$ for $x = 1, 2, \dots, 10$. Please find the following being sure to show all intermediate steps. (3 points each, 6 total)

$$\begin{aligned} (a) P(X > 8) &= P(X \in \{9, 10\}) \\ &= P(X=9 \text{ OR } X=10) \\ &= P(X=9) + P(X=10) \\ &= \frac{9}{55} + \frac{10}{55} = \frac{19}{55} = .34545 \end{aligned}$$

(b) $P(X > 8 | X \geq 2)$

$$\begin{aligned} &= \frac{P(X > 8 \text{ AND } X \geq 2)}{P(X \geq 2)} = \frac{P(X > 8)}{P(X \geq 2)} \\ &= \frac{\frac{19}{55}}{1 - \frac{1}{55}} = \frac{\frac{19}{55}}{\frac{54}{55}} = \frac{19}{54} \approx .352 \end{aligned}$$

III. Suppose that A and B are events in a sample space, S , and that $P(A) = 0.70$, $P(B) = 0.50$, and $P(A \cup B) = 0.80$. Please answer the following. For full credit on each problem you must show at least one intermediate step for any probability computations. (6 points total)

(a) Find $P(B|A)$ (3 points) $\rightarrow = \frac{P(B \cap A)}{P(A)} = \frac{.4}{.7} = \frac{4}{7} \approx .5714$

$P(A \cap B) = P(A) + P(B) - P(A \cup B) = .7 + .5 - .8 = .4$

(b) Are A and B independent events? Why or why not? (3 points)

$P(A \cap B) = .4$ A & B are not independent since $P(A \cap B) \neq P(A)P(B)$
 $P(A)P(B) = .7(.5) = .35$

IV. A store sells three brands of DVD players. The least expensive brand, B_1 , accounts for 55% of all sales and the other two brands, B_2 and B_3 , account for 25% and 20% of sales, respectively. The chances of needing a repair during the warranty period are 10% for B_1 , 5% for B_2 and 2% for B_3 . Let R be the event that a randomly selected purchaser bought a DVD player that needs a repair during the warranty period, B_1 be the event they purchased a DVD player of brand type B_1 , etc. Please answer the following. Please write all probability computations in terms of the given event names. (7 points total)

(a) Find $P(R)$. (3 points)

Law of Total

$\rightarrow P(R|B_1) = .10$
 $\rightarrow P(R|B_2) = .05$
 $\rightarrow P(R|B_3) = .02$
 $\rightarrow P(R|B_1)P(B_1) + P(R|B_2)P(B_2) + P(R|B_3)P(B_3)$
 $= .10(.55) + .05(.25) + .02(.20)$
 $= .0715$

(b) Without computing $P(B_2|R)$, describe using a complete English sentence with no abbreviations for event names the probability this represents (i.e. "This is the probability of"). (1 point)

This is the probability that given the purchased DVD player needed a repair during the warranty period, the player was of brand type B_2 .

(c) Now find $P(B_2|R)$. (3 points) (Bayes' Theorem)

$P(B_2|R) = \frac{P(B_2 \cap R)}{P(R)} = \frac{P(R|B_2)P(B_2)}{P(R)}$
 $= \frac{.05(.25)}{.0715} = .1748$