9/11/2007
Dr. Lunsford

MA371 Intro. To Prob. & Stats.
Quiz 2

Name: Solution
(20 Points Total)

I. A urn contains 4 yellow chips and 6 blue chips. A blindfolded student reaches into the urn and draws out 3 chips all at one time. Let the random variable X be the number of yellow chips among the three chips drawn. Please answer the following questions. (6 points total)

(a) What is S_X, the set of all possible values of X for this random experiment? (1 points)

$S_X = \{0, 1, 2, 3\}$

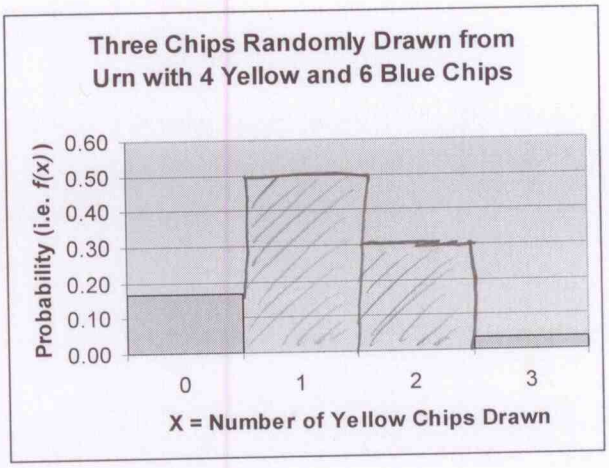
(b) Find the probability that $X = 2$, i.e. $P(X = 2)$. Please state all assumptions that justify your answer, show your answer in combinatorial form, and compute the exact value of your answer. (3 points)

Sampling w/out replacement, not counting different orders.

ways to draw 2 yellow → $\binom{4}{2}\binom{6}{1}$ ← # ways to draw 1 blue

$\binom{10}{3}$ ← # ways to draw 3 from 10

$= \frac{3}{10} = .3$



(c) To the right you are given a partially completed theoretical relative frequency histogram for X. Note that $P(X = 0) = \frac{1}{6}$ and $P(X = 3) = \frac{1}{30}$. Please complete the histogram. (2 points)

$P(X=1) = 1 - \frac{1}{6} - \frac{1}{30} - \frac{3}{10} = \frac{1}{2}$

OR

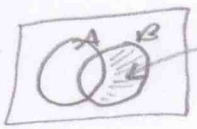
$P(X=1) = \frac{\binom{4}{1}\binom{6}{2}}{\binom{10}{3}} = \frac{1}{2}$

II. Suppose that A and B are events in a sample space, S, and that $P(A) = 0.45$, $P(B) = 0.65$, and $P(A \cup B) = 0.70$. Find the indicated probabilities. For full credit on each problem you must show at least one intermediate step. In particular you must specifically state any set equivalences you use and/or clearly show the application of any probability laws you use. Note that $A' = A^C$. (3 points each, 6 points total)

(a) Find $P(A' \cap B)$ $= P(A \cup B) - P(A)$ ← since $A \cup B = A \amalg (A' \cap B)$

$= .70 - .45 = \boxed{.25}$

$P(A \cup B) = P(A) + P(A' \cap B)$



$A' \cap B$

OR since $B = (A' \cap B) \amalg (A \cap B)$

(b) Find $P(A' \cup B')$

$P(B) = P(A' \cap B) + P(A \cap B)$

$\Rightarrow P(A' \cap B) = P(B) - P(A \cap B)$

$= .65 - .40 = \boxed{.25}$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= .45 + .65 - .70$

$= .40$

$P(A' \cup B')$ ← DeMorgan

$= P((A \cap B)') = 1 - P(A \cap B)$

$= 1 - .40 = \boxed{.60}$

III. Dr. L. was recently given a hot pink ipod. Currently she has 200 songs stored on the ipod and she likes to use the random play feature when she goes jogging with her ipod. Please answer the following. You may leave your answers in combinatorial form. (8 points total)

(a) If the random play feature *does not allow the same song to be chosen more than once*, and 50 of the songs are jazz songs, 125 of the songs are rock and roll songs, and 25 of the songs are rap songs, then what is the probability that in the first 10 songs chosen, 3 will be jazz songs, 5 will be rock and roll songs, and 2 will be rap songs? (3 points)

Sampling w/out replacement, not counting different orders

$$\frac{\binom{50}{3}\binom{125}{5}\binom{25}{2}}{\binom{200}{10}}$$

$\binom{50}{3}$ ← # ways to get 3 jazz songs; $\binom{125}{5}$ ← # ways to get 5 RnR songs; $\binom{25}{2}$ ← # ways to get two rap songs; $\binom{200}{10}$ ← # ways to choose 10 songs from 200

(b) If the random play feature *allows the same song to be chosen more than once* and if half of the songs are considered up-tempo songs and the other half are considered slow songs (i.e. choosing an up-tempo song is equally likely as choosing a slow song), then what is the probability that in the first 10 songs chosen, exactly 4 will be up-tempo songs? (3 points)

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Sampling w/ replacement, will count different orders

$$\frac{\binom{10}{4}100^4 \cdot 100^6}{200^{10}} = \frac{\binom{10}{4}}{2^{10}}$$

$\binom{10}{4}$ ← choose positions for the U songs; 100^4 ← # ways to get 4 U songs; 100^6 ← # ways to get 6 S songs; 200^{10} ← # ways to get 10 songs

Binomial Model ★ Preview of coming attraction!

(c) Again, if the random play feature *allows the same song to be chosen more than once*, then what is the probability that Dr. L. will have at least one song repeated in the first 10 randomly played songs? Hint: Think complement! (2 points)

Sampling w/ replacement, will count different orders.

$$P(\text{at least one song repeated}) = 1 - P(\text{no song repeated})$$

$$= 1 - \frac{200(199)(198)\cdots(191)}{200^{10}} = 1 - \frac{{}_{200}P_{10}}{200^{10}}$$

$200(199)(198)\cdots(191)$ ← # ways to get 10 different songs; 200^{10} ← # ways to get 10 songs