11/19/2003 MA385 Applied Prob/Stats I Name:________________
Dr. Lunsford Test 2 (100 Points Total)

Neatly show all work on this test. You may use the appendices and the front flap of your textbook. Clearly indicate your answers. Good luck!

I. Multiple Choice. Circle the best answer for each problem. (4 points each – 20 total)

1. If the random variable X has the continuous cumulative distribution function (c.d.f.)

$$F(x)=\begin{cases} 0, & x<0 \\ x^2/16, & 0\le x<4 \\ 1, & 4\le x \end{cases}$$

then $P(2\le X\le 5)=$

(a) 3/4 (b) 1 (c) 1/4 (d) 21/16 (e) None of these

2. Suppose X is $N(0,1)$. Then $P(|X|>2.5)=$

(a) .9938 (b) .0124 (c) .0062 (d) .9876 (e) None of these

3. If X is $N(0,1)$ then the value of c such that $P(|X|<c)=.99$ is

(a) 1.960 (b) 1.645 (c) 2.576 (d) 2.326 (e) None of these

4. If $X\sim b(12,.30)$ then $P(X<4)=$

(a) .7237 (b) .4925 (c) .2312 (d) .5075 (e) None of these

5. Suppose X is $N(3,9)$. Then $P(3\le X\le 6)=$

(a) .5000 (b) .3446 (c) .6554 (d) .1554 (e) None of these

II. Let X be an exponentially distributed random variable with parameter $\theta>0$. Use the moment generating function for X to show that $\mu=\theta$ and $\sigma^2=\theta^2$. (12 points)

III. In order to add variety to her cats' lives, Dr. L. randomly chooses their cat food from the pantry at feeding time. Yesterday, as she was feeding Etta Puff and Kitteney Thang, she noticed she had six cans of fish flavored cat food (salmon, tuna, etc.), and four cans of chicken flavored cat food. Dr. L. randomly chose two cans of cat food from the panty and split the contents between her two "tigers." Let the random variable X denote the number of cans of fish flavored cat food selected. Please answer the following. (13 total points total)

(a) How is the random variable X distributed? Please give the p.d.f. or p.m.f. of the distribution (with the values of any parameters) and all valid values of X. (5 points)

(c) Etta Puff particularly enjoys fish flavored cat food. What is the probability that she ate some fish flavored cat food yesterday? Write this probability in terms of the random variable X and compute the probability. (5 points)

(b) Write (<u>do not compute</u>) the probability that Dr. L. selected two cans of the same flavor cat food in terms of the random variable X. (3 points)

IV. Whiskas Choice Cuts cat food comes in packages that have a label weight of 3 ounces with a standard deviation of 0.10 ounce. Assuming the weights of these packages are normally distributed, please answer the following. (10 points total)

a. Let X denote the weight of a single package of cat food selected at random from a local grocery store. Find $P(X > 3.128)$ (6 points)

b. Suppose 15 packages are selected independently from local grocery stores and weighed. Let Y be the number of those packages that weigh more than 3.128 ounces. How is the random variable Y distributed? You should give the name of the distribution and the value of any relevant parameters including the possible values for Y. (4 points)

V. Flaws in a particular kind of metal sheeting occur randomly at an average rate of one per 10 ft^2. Let the random variable X denote the number of flaws in an 8-by-10 foot sheet. (11 points total)

(a) How is the random variable X distributed? You should give the name of the distribution, the values of any relevant parameters, and the possible values of X. (4 points)

(b) What is the probability that there will be at least two flaws in an 8-by-10 foot sheet of metal? (5 points)

(c) On average, how many flaws will an 8-by-10 foot sheet of metal have? (2 points)

VI. The random variables X and Y have the joint probability density function

$$f(x, y) = \begin{cases} 4xy, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & elsewhere \end{cases}$$

Please answer the following: (14 points total)

(a) Find the marginal probability density function for X. Neatly show all of your work. (5 points)

(b) Given that the marginal probability density function for Y is $f_2(y) = 2y$, determine if the random variables X and Y are independent. You must justify your answer. (4 points)

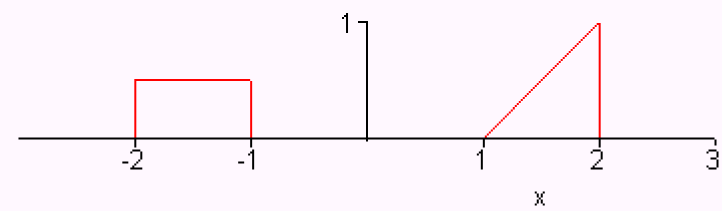
(c) Set up an integral and/or a summation to find $P(Y^2 \ge X)$. Do not compute this probability! (5 points)

VII. Let X be a random variable with probability density function

$$f(x)=\begin{cases}1/2, & -2\le x\le -1\\ x-1, & 1\le x\le 2\\ 0, & elsewhere\end{cases}.$$

A graph of the p.d.f. is shown below. Please answer the following. (20 points total)

(a) Represent the probability $P\left(\frac{-3}{2}\le X\le \frac{4}{3}\right)$ on the graph of the p.d.f. (2 points)



(b) Find the probability in part (a) by using basic geometry <u>and</u> by using the p.d.f. Clearly indicate the work for each method. (10 points)

(d) Find $E[X]$. (8 points)

<u>BONUS:</u> Only attempt if you have time! Find the cumulative distribution function (c.d.f.), $F(x)$, for X. (5 points)