2/26/2004	MA 385 Intro. to Probability	Name:________________
Dr. Lunsford	Test 1	(100 Points Total)

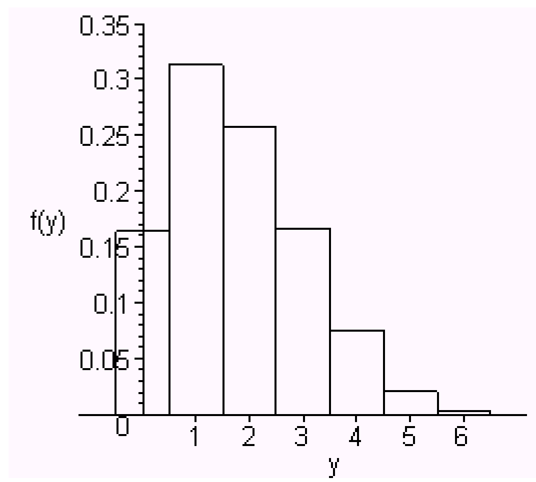
Neatly show all work on this test. Clearly indicate your answers. Unless otherwise indicated, you may leave your answers in combinatorial form. Good luck!

I. A random experiment is conducted in which a fair six-sided die is rolled and then a fair coin is flipped the number of times shown on the face of the die. Two possible outcomes from this experiment are 4HHTH and 3THT. Let the random variable X denote the face value on the die for an outcome and the random variable Y denote the number of heads for an outcome. Below you are given a graph of the probability mass function (p.m.f.) for Y and a table that contains the values of the p.m.f. for Y. Please answer the following questions. (32 points total)

y	0	1	2	3	4	5	6
$f(y) = P(Y = y)$	.16406	.3125	.25782	.16667	.07552	.02083	.0026

(a) How many outcomes are possible for this random experiment? (4 points)

(b) Find the mean of the distribution for the random variable Y. Show the formula you use and how you are plugging the data into the formula to compute this value. (6 points)



(c) Suppose we run the experiment 20 independent times and observe the following values of Y: 1, 1, 1, 3, 0, 2, 2, 3, 4, 2, 2, 2, 1, 1, 0, 3, 1, 2, 1, 4
Find the relative frequencies for this data and plot the corresponding histogram on the same axes as the p.m.f. of Y. Please shade your histogram rectangles so that I can distinguish your histogram from the p.m.f. of Y. (5 points)

(d) Find the mean of the data in part (c). Show the formula you use and how you are plugging the data into the formula to compute this value. (5 points)

I. Continued.

(e) If $X = 5$, what is the actual (not empirical!) probability that $Y = 4$? Write this probability using the random variables X and Y. (6 points)

(f) Find $P(Y = 4)$ (Note: Find the actual, not empirical probability). Hint: Condition on the relevant values of X. Be sure to write all probabilities you use in terms of the random variables X and Y. (6 points)

II. Suppose P is a probability function on a sample space S and A and B are events in S such that $P(A) = 0.52$, $P(B) = 0.45$, and $P(A \cap B) = 0.234$. Please answer the following. You must show one intermediate step on each computation for full credit. (5 points each, 30 points total)

a. $P(A \cup B)$

b. $P(B \mid A)$

c. $P(A \cap B^C)$

d. $P(A^C \cup B^C)$

e. Find the probability that event A or event B but not both occur.

f. Are the events A and B independent events? Why or why not?

III. An urn contains 8 red and 10 white balls. Find the probability that the 6^{th} ball drawn is the 4^{th} white ball drawn if:

(a) the balls are drawn without replacement. (5 points)

(b) the balls are drawn with replacement. (5 points)

IV. Urn A has 10 red and 10 white balls and Urn B has 6 red and 14 white balls. Two balls are drawn from each urn without replacement. Find the probability that all four balls are the same color. (8 points)

V. Show that if the events A and B are independent, then the events A and B^C are also independent. (8 points)

VI. At a certain small college, 20% of the students are getting a major in the college of liberal arts, 30% are getting a major in the sciences, and 50% are getting a major in education. 60% of the liberal arts students are honor students, 40% of the science majors are honor students, and 80% of the education majors are honor students. Choose a student at random from the population of students at this college and let L be the event the student is getting a major in liberal arts, S the event the student is getting a major in the sciences, E the event the student is getting a major in education, and H the event the student is an honor student. (12 points total)

(a) If the randomly selected student is an honor student, what is the probability that the student is majoring in the sciences? To receive full credit you must write <u>all probabilities</u> that you use for this computation in terms of the event names given above. (10 points)

(b) Compare the probability computed in part (a) to the probability that a randomly selected student from the college is majoring in the sciences. Explain why the numerical difference in these two probabilities makes sense. (2 points)