10/1/2003 MA 385 Intro. to Probability Name:________________
Dr. Lunsford Test 1 (100 Points Total)

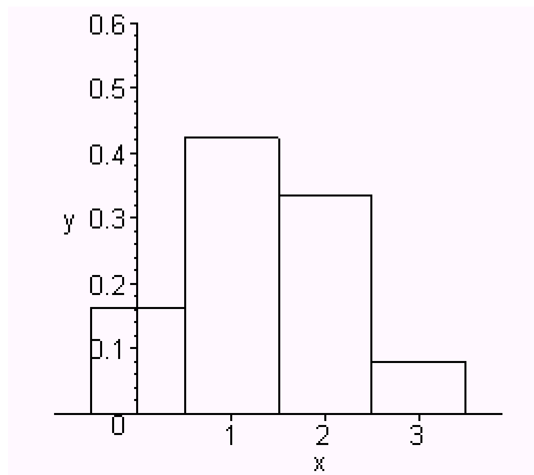
Neatly show all work on this test. Clearly indicate your answers. Unless otherwise indicated, you may leave your answers in combinatorial form. Good luck!

I. A bag of Halloween candy contains 20 miniature Snicker bars, 15 miniature Reeses Cups, and 10 Tootsie Rolls. A trick-or-treater reaches into the bag and pulls out 3 pieces of candy. Let the random variable $X$ denote the number of Snicker bars among the 3 pieces. Below you are given a graph of the probability mass function (p.m.f.) for $X$.

(a) Suppose we take a random sample of size twenty from the distribution of $X$ (i.e. we repeatedly perform the experiment 20 independent times and observe the value of $X$ on each run of the experiment. Note this implies we are returning the three pieces of candy to the bag for each run of the experiment). The following values for $X$ are observed:

0, 2, 1, 0, 1, 1, 1, 2, 0, 1, 2, 1, 0, 1, 1, 2, 1, 1, 2, 1

Find and plot the empirical relative frequency histogram on the same axes as the p.m.f. of $X$. (6 points)




(b) Find the sample mean of the data in part (a). You are welcome to use technology here however please show the formula you use and how you are plugging the data into the formula to compute this value. (4 points)

(c) Find the sample standard deviation of the sample data in part (a). Again you are welcome to use technology here however please show the formula you use and how you are plugging the data into the formula to compute this value. (5 points)

(d) What is the actual (not empirical!) probability that $X = 2$? (5 points)

(e) What is the probability that all three pieces of candy will be of the same type? (5 points)

II. Suppose $P$ is a probability function on a sample space $S$ and $A$ and $B$ are events in $S$ such that $P(A) = 0.55$, $P(B) = 0.45$, and $P(A \cap B) = 0.25$. Please answer the following. (4 points each, 24 points total)

a. $P(A \cup B)$

b. $P(B \mid A)$

c. $P(A \cap B^C)$

d. $P(A^C \cup B^C)$

e. Find the probability that event A or event B but not both occur.

f. Are the events $A$ and $B$ independent events? Why or why not?

III. A fair coin is flipped ten times. Let the random variable $X$ be the number of heads in the ten flips. What is the probability of getting exactly eight heads in the ten flips given that at least one head was obtained in the ten flips? (7 points)

IV. Show that if the events $A$ and $B$ are independent, then the events $A^C$ and $B^C$ are also independent. (8 points)

V. Suppose there are fourteen songs on a CD and you like eight of them. Being a fun-loving probability student, you use your random play button on your CD player to play the songs on the CD. Your CD player's random play button is programmed so that once a randomly chosen song has been played, it will not be played again. Please answer the following (5 points each – 25 points total)

(a) Find the probability that you will like the first song played, not like the second song played, and like the third song played.

(b) Find the probability that you will like at least one of the first three songs played.

(c) Find the probability that the fourth song played is only the second song (so far) that you like.

(d) Now suppose that you borrow a friend's CD player and that her CD player's random play button is programmed so that a randomly chosen song can be played more than once. What is the probability that you will like the first song played, not like the second song played, and like the third song played?

(e) Under the same assumptions as part (d), find the probability that you will like two of the first three songs played.

VI. A biased coin, twice as likely to come up heads a tails, is tossed once. If it shows heads, a chip is drawn from Urn 1, which contains three white and four red chips; if it shows tails, a chip is drawn from Urn 2, which contains six white and three red chips. Please answer the following. (11 points total)

(a) If a white chip was drawn, what is the probability that the coin came up tails? Clearly identify all names of events and probabilities using the event names for this computation. (9 points)

(b) Compare the probability computed in part (a) to the probability that the coin flip is a tail. Explain any numerical differences in these two probabilities. (2 points)