4/1/2008 MATH405 Numerical Analysis Name:_____
Dr. Lunsford Quiz 4 (30 Points Total)

Please show all your work on separate sheets of paper. Please show any calculator approximations to the accuracy shown on your calculator display.

1. We desire to fit the data set $(x_i, y_i), i = 1..m$ with the model $y = be^{ax}$ for some unknown a and b. Using least squares, we linearize the equation to get $y = \hat{b} + ax$ where $y = \ln y$ and $\hat{b} = \ln b$. We then apply our least squares algorithm to get the solution $y = 0.5 - 4x$. Please answer the following: (9 points total)

   (a) What is the model $y = be^{ax}$? (4 points)

   (b) When using least squares we minimize the sum of the squares of the residuals. Clearly write the sum of the squares of the residuals that was minimized for this problem. You must use the original variables (i.e. do not use any variable with a "hat" on it) when writing this sum of squares. (5 points)

2. Suppose you are given the function values for some function at the x coordinates 1.05, 1.10, 1.15, 1.20, 1.25, and 1.30. Please answer the following questions. (9 points total)

   (a) Please list all possible formulas you may use from formulas 4.1, 4.4, 4.5, 4.6, and 4.7 to compute $f'(1.25)$. You should give the formula number and the value of h used for the formula. (6 points)

   (b) Without performing the computation, which of the approximations in part (a) do you expect to be the most accurate? Why? (3 points)

3. What values of n and h are required to compute $\int_0^{1.5} \sin(x)dx$ using the Composite Simpson's Rule with an absolute error of no more than 10^{-6}. Note: you should try to find the smallest value of n (and hence the largest value of h). (8 points)

4. Consider the ODE:

$$\frac{dy}{dt} = \frac{t\sin(t^2)}{y} \text{ for } 0 \le t \le 4,\ y \ge 0$$

with initial condition $y(0) = \frac{1}{2}$. (9 points total)

   (a) Solve for y. (6 points)

   (b) To your right are given a direction field for the differential equation. Draw your solution on the direction field. (3 points)