2/27/2008 MATH405 Numerical Analysis Name:_____
Dr. Lunsford Quiz 3 (30 Points Total)

Please show all your work on separate sheets of paper. Please show all calculator approximations to the accuracy shown on your calculator display.

1. Use the equation $x-1.3-0.2\cos(x)=0$ to answer the following questions. Please write all approximations to the accuracy of your calculator display. (19 points total)

(a) Complete <u>three iterations</u> of the Bisection method with starting interval $0,2$ to find an approximate solution of the equation. Neatly show all of your work. Note: You may want to organize your work in a table. (5 points)

(b) If you stop at iteration three of the bisection method, what is your approximate solution of the equation? Clearly indicate your answer. (1 point)

(c) What is the *maximum possible* absolute error for this approximation (do not compute the actual error)? (2 points)

(d) Given that $x=1.344812966$ is an approximate solution accurate to 9 decimal places, what is the absolute error and the relative error for your approximate solution? Clearly indicate your answers. (2 points)

(e) How many significant digits of accuracy does your approximate solution to the equation have? <u>You should justify your answer using one of the errors computed in part (d)</u>. (1 point)

(f) What is the minimum number of iterations of the bisection method required to guarantee the approximate solution of the equation will have an absolute error of no more than 10^{-5}? Assume the same starting interval as above. (4 points)

(g) Perform two iterations of Newton's Method to approximate a solution to the equation with $x_0=1$ (i.e. only compute x_1 and x_2). (4 points)

II. Let $f(x)=3^{-x}$. Please answer the following: (6 points total)

(a) Give the unsimplified form of the Lagrange polymomial for f that passes through the nodes with x-coordinates $x_0=0$, $x_1=1$, and $x_2=2$. (4 points)

(b) Use the Lagrange polynomial computed in part (a) to approximate $\frac{1}{\sqrt[3]{3^2}}$. (2 points)

III. Suppose a Taylor polynomial of degree 2 centered at $x=1$ is used to approximate $f(x)=3^{-x}$ on the interval $[0,2]$. Find an upper bound for the maximum error for all $x\in[0,2]$. Hint: Recall $3^{-x}=e^{\ln(3^{-x})}$ (5 points)