3/3/2003 MA423 Numerical Analysis Name:_____
Dr. Lunsford Quiz 5 (20 Points Total)

Neatly show ALL of your work and CLEARLY indicate your answers. Use the back of the page if necessary.

I. (a) Use the nodes $x_0 = \frac{\pi}{4}$, $x_1 = \frac{\pi}{2}$, and $x_2 = \frac{3\pi}{4}$ to find a Lagrange polynomial that approximates $\sin(x)$. DO NOT simplify your answer! (6 points)

(b) Find an error bound for the absolute error if you use the polynomial in part (a) above to approximate $\sin\left(\frac{3\pi}{8}\right)$? Remind me to put the remainder theorem for Lagrange polynomials on the board for you! (3 points)

(c) Find an error bound for the absolute error if you use the polynomial in part (a) above to approximate $\sin(x)$ on the interval $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$? (4 points)

II. A natural cubic spline for a function f is defined on $[0,2]$ by

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3, & 0 \le x < 1 \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3, & 1 \le x \le 2 \end{cases}$$

Find b, c, and d. (7 points)