3/5/2003 MA423 Numerical Analysis Name:_____
Dr. Lunsford Mid-Term Exam (100 Points Total)

Neatly show all of your work on this test. If you need to use extra paper, please indicate so and attach to this test. Good luck!

I. Compute the expression $(\frac{1}{3}-\frac{3}{11})\frac{1029}{3}$ using:

(a) three digit rounding arithmetic. (5 points)

(b) three digit chopping arithmetic. (5 points)

(c) Find the exact value of the expression to the accuracy of your calculator. Use this value to determine the absolute and relative errors for the three digit rounding computation. To how many significant digits is the three digit rounding computation accurate? Clearly indicate your answers! (9 points)

II. Find $\lim_{n\to\infty}\left(\cos\left(\frac{1}{n}\right)+\frac{1}{2n^2}\right)$ and find the order of convergence of the limit. (10 points)

III. Let $f(x) = x^3 + 3x^2 - 1$. Use the Newton-Raphson Method with $p_0 = -1$ to find $p_1$ and $p_2$. (10 points)

IV. Consider the equation $x^3 + x = 5$.

(a) Explain why the equation has a solution on the interval $[1,3]$. (5 points)

(c) Explain how you would use the Bisection Method to find a solution to this equation. (i.e., what function are you finding a zero of, what is your starting interval, etc.). (4 points)

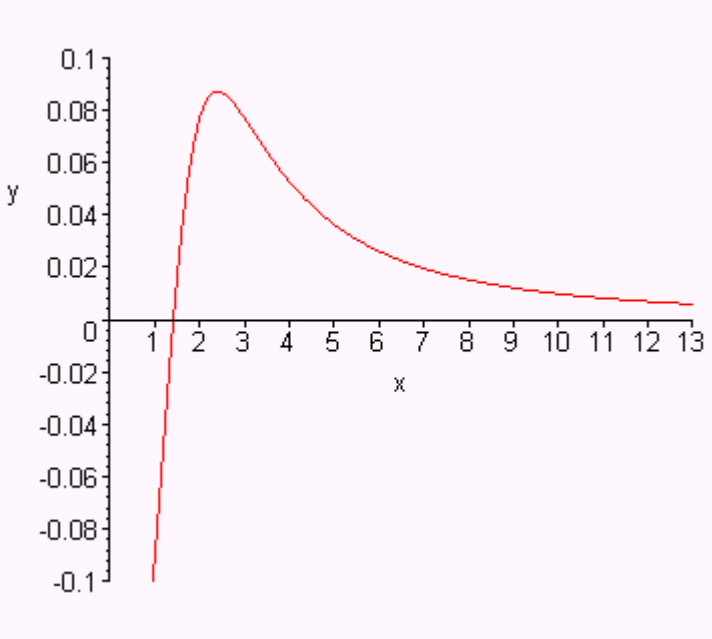
(b) How many iterations are necessary to approximate a solution to the equation via the Bisection Method using the starting interval $[1,3]$ such that the absolute error is guaranteed to be no more than $10^{-9}$? Show all work to justify your answer. (6 points)

V. A clamped cubic spline $S$ is defined on $[0,2]$ by

$$S(x)=\begin{cases} S_0(x)=1+Bx+2x^2-2x^3, & 0\le x<1 \\ S_1(x)=1+b(x-1)-4(x-1)^2+7(x-1)^3, & 1\le x\le 2 \end{cases}$$

Find $f'(0)$ and $f'(2)$. Neatly show all work to find these values. (10 points)

VI. Use the graph below and the point $p_0=4$ to graphically find $p_1$ and $p_2$ using Newton's Method. Will the sequence $p_n$ converge to the zero between 1 and 2? If not, what would be a better $p_0$ if you want the Newton iterates to converge to the zero between 1 and 2? (8 points)



VII. Consider the function $f(x) = \cos x$.

(a) What is the $4^{th}$ degree Taylor Polynomial centered at zero for $f$? Use this polynomial to approximate $\cos\left(25^\circ\right)$. (6 points)

(b) What is the actual absolute error for the approximation of $\cos\left(25^\circ\right)$ in part (a)? Note: Use your calculator approximation as the actual value of $\cos\left(25^\circ\right)$. (4 points)

(c) Find an error bound for the absolute error when using the $4^{th}$ degree Taylor Polynomial in part (a) to approximate $\cos 25^\circ$. (6 points)

(d) Now use the nodes $x_0 = \frac{-\pi}{4}$, $x_1 = 0$, and $x_2 = \frac{\pi}{4}$ to find a Lagrange polynomial that approximates $\cos x$. DO NOT simplify your answer. (6 points)

(e) Find an error bound for the absolute error when using the Lagrange polynomial in part (d) to approximate $\cos x$ on the interval $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$. (6 points)