10/25/2000 MA423 Numerical Analysis Name:____________
Dr. Lunsford Mid-Term Exam (100 Points Total)

Neatly show all of your work on this test. If you need to use extra paper, please indicate so and attach to this test. Good luck!

I. Compute the expression $(\frac{1}{3}-\frac{3}{11})\frac{100}{3}$ using:

(a) four digit rounding arithmetic. (5 points)

(b) four digit chopping arithmetic. (5 points)

II. Use the a Taylor polynomial for e^x about $x=0$ to show that $\lim_{h\to 0}(e^{h^2}-h^2-1)=0$ converges with order h^4 as h goes to zero. (8 points)

III. Let $f(x)=-x^3-\cos x$. Use the Newton-Raphson Method with $p_0=-1$ to find p_1 and p_2. (10 points)

IV. Let $f(x) = 4 - x - x^3$ on the interval $[1,3]$.

(a) Complete the following table using the Bisection Method. (10 points)

n	a	b	p	Sign of $f(p)$
1	1	3	2	$f(p) < 0$
2				
3				
4				

(b) How many iterations are necessary to approximate a zero for f using the starting interval $[1,3]$ such that the absolute error is guaranteed to be no more than 10^{-9}? (5 points)

V. A natural cubic spline S is defined on $[0,2]$ by

$$S(x) = \begin{cases} S_0(x) = 2 + 3x - x^3, & 0 \le x < 1 \\ S_1(x) = a + b(x-1) + c(x-1)^2 + d(x-1)^3, & 1 \le x \le 2 \end{cases}$$

Find a, b, c, and d. (10 points)

VI. Consider the expression $\sum_{n=1}^{5} n!$.

(a) How many additions and multiplications are required to compute the expression as it is written? (3 points)

(b) Note that the expression can be written in a nested multiplication format as follows:

$$\sum_{n=1}^{5} n! = 1 + 2(1 + 3(1 + 4(1 + 5)))$$

How many additions and multiplications are required to compute the expression using the nested multiplication format? (3 points)

(c) Write a pseudo code algorithm to compute the expression $\sum_{n=1}^{k} n!$ using nested multiplication. You may assume that your only input to the algorithm is the value of the integer k. (8 points)

VII. Suppose that $f \in C^{(5)}[1.0,1.8]$ and $|f^{(5)}(x)| \le 10$ for all $x \in [1.0,1.8]$. Below is a table generated using Neville's Method for approximating f with a LaGrange polynomial of degree 4 or less at the point $x = 1.65$. Note that the first column corresponds to the input values x_0 through x_4 and the second column corresponds to f evaluated at x_0 through x_4.

1.00000000	.76519770				
1.30000000	.62008600	.45078902			
1.60000000	.45540220	.42795490	.42605206		
1.70000000	.28181860	.36861040	.37602846	.37960158	
1.80000000	.11036230	.36754675	.36834449	.37064968	.37232816

(a) What is the approximation of $f(1.65)$ using the LaGrange polynomial of degree at most four? Hint: You should NOT have to crank and grind to get this approximation! (2 points).

(b) Write out (DO NOT SIMPLIFY!) the LaGrange polynomial of degree at most two generated by the nodes $(x_0, f(x_0))$, $(x_1, f(x_1))$, and $(x_2, f(x_2))$. (9 points)

(c) What is the approximation of $f(1.65)$ using the LaGrange polynomial generated in part (b). Hint: You should NOT have to crank and grind to get this approximation! (2 points).

(d) Find an error bound for $|L(1.65) - f(1.65)|$ where L is the LaGrange polynomial approximation of f of at most degree four generated by the nodes in the above table. (5 points)

(e) Find an error bound for $|L(x) - f(x)|$ with $x \in [1.0,1.8]$ where L is the LaGrange polynomial approximation of f of at most degree four generated by the nodes in the above table. (5 points)

VII. Suppose you are asked to analyze simulation data predicting the flight of a missile. Part of the analysis is to determine how well the simulation predicts the maximum altitude reached by the missile. Suppose you are given 100 data points each consisting of a time (in seconds) and an altitude (in meters). How could you use the algorithms we have investigated in this class to help you determine the maximum altitude of the missile? You should compare and contrast the various methods you are considering using and you should also give a rationale as to why you would use one method over another (for instance, would you approximate the missile's altitude function using LaGrange polynomials or using a cubic spline?). (10 points)