4/24/2006	MATH405 Numerical Analysis	Name:____________
Dr. Lunsford	Test 2	(100 Points Total)

You have been provided a data file called Data_for_Test2.xls. Please resave this file with the syntax YourNameTest2.xls. Please email this file to me when you hand in your exam. Neatly show all of your work for this test on separate paper (which I will provide). Clearly indicate if and how you use any of the available software packages (Derive, MATLAB, Excel) to solve the problem. Since you are not emailing me Derive or MATLAB output, please give me the complete command you use, including your exact input, on any problem in which you use either of these packages. Also clearly indicate any theorems or formulas you use in the text with both the number of the formula or theorem number and the page number of which it is found. Lastly, please show any calculator approximations to the accuracy of the display on your calculator. Good luck!

I. The distance (in feet) traveled by an object is given by the following table (time is in seconds). This data is also available to you on the Excel worksheet for the test. Please answer the following questions. (25 points total)

Time (s)	Distance (ft)
8	17.453
9	21.46
10	25.752
11	30.301
12	35.084

(a) List all possible ways that three-point formulas may be employed to find the velocity of the object at time $t = 10$ seconds. You should give the formula number in your textbook as well as the appropriate value of h. (6 points)

(b) Use an appropriate three-point formula to find the velocity of the object at time $t = 12$ seconds. (6 points)

(c) Use the five point formula numbered (4.6) to find the velocity of the object at time $t = 10$ seconds. (6 points)

(d) The data in the table were generated by the function $d(t) = -70 + 7t + 70e^{-t/10}$. Find a bound for the absolute error for the computation in part (c). Compare this to the actual absolute error. (7 points)

II. Consider the definite integral $\int_0^2 xe^{-x}dx$. Please answer the following questions. (25 points total)

(a) Find the minimum number of subintervals needed to approximate the integral using the composite trapezoid rule with an absolute error of no more than 5×10^{-6}. (7 points)

(b) Repeat part (a) but use the composite Simpson's rule instead. (6 points)

(c) Approximate the integral to the desired degree of accuracy in part (a) using the composite trapezoid rule. (6 points)

(d) Find the actual value of the integral and compute the actual absolute error for the approximation in part (c). (6 points)

<u>III</u>. The logistic curve is used for population growth models. It has the form $y = \dfrac{L}{1 + Ce^{At}}$ where L is some constant and A and C are parameters usually determined from the data. In the table below and in your Excel spreadsheet you are given the world population in millions from 1950 to 2000*. Please answer the following questions. (25 points total)

Year	Population (in millions)
1950	2555
1955	2780
1960	3040
1965	3346
1970	3708
1975	4087
1980	4454
1985	4850
1990	5276
1995	5686
2000	6079

(a) To use least-squares to fit data to the logistics curve you must transform the equation. Show that the transformed equation has the following form: $Y = AX + B$ where $Y = \ln\left(\dfrac{L}{y} - 1\right)$, $X = t$, and $B = \ln C$. (5 points)

(b) Use the least-squares fit of the data in the Excel spreadsheet to the logistics curve to find A and C. Assume $L = 12000$. Clearly explain your approach. (5 points)

(c) Use the logistic model found in part (b) to predict the world population for the year 2005*. (5 points)

(d) Find the sum of the squares of the residuals (errors) for the model. (5 points)

(e) Is the sum of the squares of the residuals the smallest possible value for a logistic fit to the data? Why or why not? (5 points)

<u>IV.</u> The differential equation $\dfrac{dp}{dt} = kp$ can be used to model a population where k is the growth rate of the population. This model assumes a constant growth rate that does not take into account the size of the population. If we let $k = k_{\max}(1 - p/p_{\max})$ where $p_{\max}$ is a constant that equals the maximum population (also sometimes called the carrying capacity) and $k_{\max}$ is a constant that equals the growth rate of the population under unlimited conditions, then we get a new differential equation:

$$\frac{dp}{dt} = k_{\max}(1 - p/p_{\max})p \tag{1}$$

The exact solution of this equation is given by $p = \dfrac{p_{\max}}{1 + \left(\dfrac{p_{\max} - p_0}{p_0}\right)e^{-k_{\max}t}}$ where p_0 is the initial population. Note that this is a logistic curve. Please answer the following questions. (25 points total)

(a) Explain how letting $k = k_{\max}(1 - p / p_{\max})$ takes into account the size of the population in terms of population growth. (6 points)

(b) Show that the right hand side of the differential equation (1) satisfies a Lipschitz condition in p for all time and for $0 \le p \le p_{\max}$. Clearly identify the Lipschitz constant. (6 points)

(c) Use the Runge Kutta method of order four to approximate a solution to (1) from 1950 to the year 2005 with $p_{\max} = 12000$ million and p_0 (in 1950)=2555 million people. Use a step size of 5 years. What is your prediction for the world population for the year 2005? (7 points)

(d) Find the sum of the squares of the residuals (errors) for the Runge Kutta solution of the model and the population data in the Excel spreadsheet. (6 points)

*The data in the table is from the textbook, “Applied Numerical Methods with MATLAB for Engineers and Scientists” by Steven C. Chapra , McGraw Hill, 2005. According to the US Census Bureau, the total world population for the year 2005 was 6451 million people (http://www.census.gov/ipc/www/worldpop.html).