3/6/2008	MA405 Numerical Analysis	Name:____________
Dr. Lunsford	Mid-Term Exam	(100 Points Total)

Neatly show all of your work on separate paper. Attach this test to your work. Good luck!

I. Given the expression $\left(\frac{1}{6}-\frac{2}{13}\right)\frac{27783}{3}$ please answer the following. (11 points total)

(a) Compute the expression using four digit chopping arithmetic. (5 points)
(b) Find the exact value of the expression to the accuracy of your calculator. Use this value to determine the absolute and relative errors for your four digit chopping computation. To how many significant digits is the three digit chopping computation accurate? Clearly indicate your answers! (6 points)

II. Consider the sequence $x_n = n\left(1-\cos\left(\frac{1}{n}\right)\right)$. Given that $x_n \to 0$ as $n \to \infty$ find the order of convergence of the sequence. (6 points)

III. Consider the equation $\cos(x) = \sqrt{x}$. Please answer the following questions. (18 points total)

(a) Explain why the equation has a solution on the interval $\left[0, \frac{\pi}{2}\right]$. (5 points)
(b) Explain how you would use the Bisection method to find a solution to this equation. (i.e., clearly identify the function, $f(x)$, for which you are finding a zero and what would be your starting interval). DO NOT perform the bisection method. (4 points)
(c) Explain how you would use Newton's Method to find a solution to this equation (i.e., clearly identify the function, $f(x)$, for which you finding a zero, what would be your initial starting point for Newton's method, and what would be your function, $g(x)$, for iterating in Newton's method (i.e. for which you are finding a fixed point)). DO NOT perform Newton's method. (5 points)
(d) Explain how you could use fixed point iteration (but not Newton's Method!) to find a solution to this equation (i.e., clearly identify the function, $g(x)$, for which you are finding a fixed point (this should not be the same $g(x)$ you used for Newton's Method above), and what would be your initial starting point for the fixed point iteration). DO NOT perform fixed point iteration. (4 points)

IV. Suppose you are given the information about a certain function $f \in C^3[0,2]$ in the table to your right. Please answer the following: (8 points each, 16 points total)

x	$f(x)$	$f'(x)$	$f''(x)$
0	2.1	-0.6	1.3
1	3.2		
2	1.8		

(a) Using the best method you can think of, estimate $f(0.10)$. Why did you choose this method?
(b) Using the best method you can think of, estimate $f(1.9)$. Why did you choose this method?

V. A natural cubic spline S is defined on $[1,3]$ by

$$S(x) = \begin{cases} S_0(x) = 1 + \frac{1}{4}(x-1) - \frac{1}{4}(x-1)^3, & 1 \le x < 2 \\ S_1(x) = 1 + a(x-2) + b(x-2)^2 + c(x-2)^3, & 2 \le x \le 3 \end{cases}$$

Find *a*, *b*, and *c*. Neatly show all work to find these values. (8 points)

VI. Consider the expression $\sum_{i=1}^{5} \frac{1}{i!}$. Please answer the following. (14 points total)

(a) How many floating point operations (additions/subtractions, multiplications/divisions) are required to compute the expression as it is written? (3 points)

(b) Note that the expression can be written in a nested multiplication format as follows:

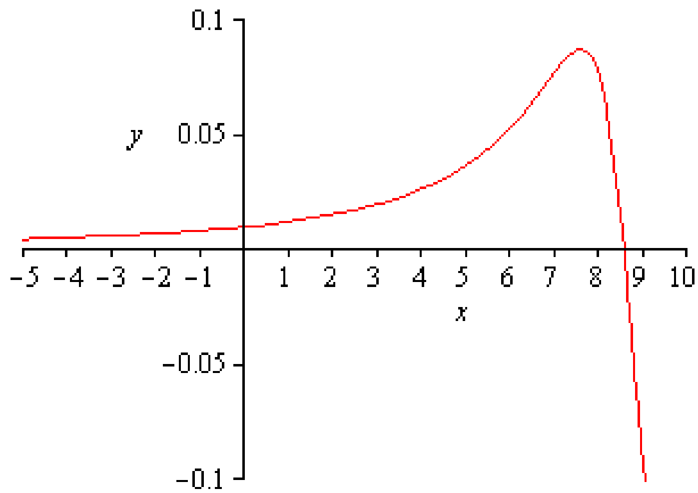
$$\sum_{i=1}^{5} \frac{1}{i!} = 1 + \frac{1}{2}(1 + \frac{1}{3}(1 + \frac{1}{4}(1 + \frac{1}{5})))$$

How many floating point operations are required to compute the expression using the nested multiplication format? (3 points)

(c) Write a pseudo code algorithm to compute the expression $\sum_{i=1}^{n} \frac{1}{i!}$ using nested multiplication.

You may assume that your only input to the algorithm is the value of the integer n. Your algorithm should perform the same number of floating point operations as part (b) when $n = 5$. (8 points)

VII. Use the graph to your right and the point $p_0 = 4$ to graphically find p_1 and p_2 using Newton's Method. Clearly indicate your answers. Will the sequence p_n converge to the zero between 8 and 9? If not, what would be a better p_0 if you want the Newton iterates to converge to the zero between 8 and 9? (5 points)



VIII. Given the function $f(x) = (1+x)^{(1/3)}$
please answer the following: (22 points total)

(a) Find the 3rd degree Taylor polynomial centered at zero for f. (8 points)
(b) Use the polynomial found in part (a) to approximate the cube root of 1.5. (4 points)
(c) Use the error formula in Taylor's theorem to find an upper bound for the error in your approximation in part (b). (6 points)
(d) To how many significant digits is your estimate in part (b) accurate? Use your calculator approximation of the cube root of 1.5 as the actual value. (4 points)