Pledge:

2/24/2010
Dr. Lunsford

MATH361 Calculus III
Test 1

Name: Solution
(100 Points Total)

Please show all work on this test.

Problem I. Find the angle (in degrees) between the two vectors $\mathbf{v} = \langle 2,2,1 \rangle$ and $\mathbf{u} = \langle 1,2,2 \rangle$. (5 points)

$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{8}{9} \therefore \theta = 27.27°$

Problem II. Consider the points $A = (-2,1,0)$, $B = (3,3,-1)$, and $C = (0,2,0)$. Please answer the following: (14 points total)

(a) Find the equation of the plane through the three points. (10 points)

$\overrightarrow{AB} = \langle 5,2,-1 \rangle$ $\quad \overrightarrow{AB} \times \overrightarrow{AC} = \left| \begin{matrix} i & j & k \\ 5 & 2 & -1 \\ 2 & 1 & 0 \end{matrix} \right| = \langle 1,-2,1 \rangle$

$\overrightarrow{AC} = \langle 2,1,0 \rangle$

$1(x-0) - 2(x-2) + 1(z-0) = 0$

$x - 2y + z = -4$

(b) Find the area of the triangle enclosed by the three points. (4 points)

$\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}| = \frac{1}{2}|\langle 1,-2,1 \rangle| = \frac{\sqrt{6}}{2}$

Problem III. Consider the two lines given by the following parametric equations:

Line 1: $x = 3-t$, $y = 2+t$, $z = 1-t$
Line 2: $x = 3+2s$, $y = -2-s$, $z = 5+s$

Determine if the two lines are parallel, intersecting, or skew. (10 points)

slope vector line 1: $\langle -1,1,-1 \rangle$ } Not constant multiples of
slope vector line 2: $\langle 2,-2,1 \rangle$ } each other is not parallel

See if lines intersect:

$3-t = 3-2s \Rightarrow s = -\frac{t}{2}$

$2-t = -2-s \Rightarrow 2+t = -2+\frac{t}{2} \Rightarrow \frac{t}{2} = -4 \Rightarrow t = -8$

$\Rightarrow s = 4$

check w/ z: $1-(-8) = 9$, $5+4 = 9$

point of intersection: $(11,-6,9)$

$\perp$

Problem IV. Let $\mathbf{u} = \langle 3, 4x+y+1, 4 \rangle$ and $\mathbf{v} = \langle x+y, 2, y \rangle$. Find values of x and y such that $\mathbf{u}$ and $\mathbf{v}$ are orthogonal. Note that there is more than one possible answer here! Check to make sure the two values of x and y you find are correct. (10 points)

$\vec{u} \cdot \vec{v} = 0 \Rightarrow 3(x+y) + 2(4x+y+1) + 4y = 0$

$\Rightarrow 11x + 9y = -2$ Let $x = -1, y = 1$ then

$\vec{u} = \langle 3, -2, 4 \rangle$, $\vec{v} = \langle 0, 2, 1 \rangle$ and

$\vec{u} \cdot \vec{v} = 0 - 4 + 4 = 0$

Problem V. Suppose the velocity vector for an object moving in space is $\mathbf{v}(t) = \langle t^2, e^{2t}, \sin(t) \rangle$ where t is in seconds and $\mathbf{v}(t)$ is in feet per second for each component. Please answer the following: (10 points total)

(a) Find the position vector, $\mathbf{r}(t)$, for the object if $\mathbf{r}(0) = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$. (6 points)

$\vec{r}(t) = \langle \frac{1}{3}t^3 + C_1, \frac{1}{2}e^{2t} + C_2, -\cos t + C_3 \rangle$

$\vec{r}(0) = \langle C_1, \frac{1}{2} + C_2, -1 + C_3 \rangle = \langle 1, 4, -1 \rangle$

$\Rightarrow C_1 = 1, C_2 = \frac{7}{2}, C_3 = 0$

$\therefore \vec{r}(t) = \langle \frac{1}{3}t^3 + 1, \frac{1}{2}e^{2t} + \frac{7}{2}, -\cos t \rangle$

(b) Find the acceleration vector for the object at time $t = 0$. (4 points)

$\vec{a}(t) = \vec{v}'(t) = \langle 2t, 2e^{2t}, \cos(t))$

$\vec{a}(0) = \langle 0, 2, 1 \rangle$

Problem VI. Consider the surface $z = f(x, y) = x\ln(x^2 - y^3) + 3xy$. Please answer the following: (12 points total)

(a) Find $f_x(x, y)$ and $f_y(x, y)$. DO NOT simplify your answers! (8 points)

$f_x(x,y) = \ln(x^2 - y^3) + \frac{x}{x^2 - y^3} \cdot 2x + 3y$

$f_y(x,y) = \frac{x}{x^2 - y^3}(-3y^2) + 3x$

x fixed
y varies

(b) Find the slope of the tangent line to the curve of intersection of the surface and the plane $x = 3$, at the point $(3, 2, 36)$. (4 points)

$f_y(3,2) = \frac{-3(3)(4)}{1} + 3(3) = -27$

Problem VII. If $z = x^2 + xy^3$, $x = uv^2 + w^3$ and $y = u + ve^w$ use the Chain Rule to find $\dfrac{\partial z}{\partial u}$ when $u = 2$, $v = 1$, and $w = 0$. (8 points) $\Rightarrow x=2, y=3$

$$\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u} = (2x+y^3)(v^2) + 3xy^2(1)$$

So
$$(4+27)(1) + 3(2)(9)$$
$$31 + 54 = 85$$

Problem VIII. For each limit below determine if the limit exists or not. Justify your answers. (5 points each – 10 total)

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+2y^2}$ $\therefore$ DNE

x-axis: $(y=0)$: $\dfrac{0}{x^2} \to 0$ $x=y$: $\dfrac{2x^2}{3x^2} \to \dfrac{2}{3}$

y-axis $(x=0)$: $\dfrac{0}{2y^2} \to 0$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^3}{x^2+2y^2}$ $= 0$

$$0 \le \left|\frac{3x^3}{x^2+2y^2} - 0\right| = \left|\frac{3x^3}{x^2+2y^2}\right| \le \left|\frac{3x^3}{x^2}\right| \le 3|x|$$

as $x \to 0$ $3|x| \to 0$ $\therefore$ by the squeezing theorem

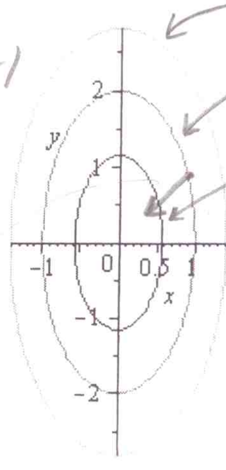
Problem IX. To your right you are given the level curves for the function $f(x,y) = -x^2 - \dfrac{y^2}{4}$ for the levels $-\dfrac{1}{3}$, -1 and -2. Please answer the following: (6 points total)

-2

-1

$-\frac{1}{3}$

$\to -x^2 - \frac{y^2}{4} = -1$
$y=0 \Rightarrow$
$x=1$



(a) Clearly label which curve is for each of the three levels given (i.e. match the curve to its level). (4 points)

$-x^2 - \frac{y^2}{4} = -\frac{1}{3} \to y=0 \Rightarrow x = \sqrt{\frac{1}{3}}$

(b) If you are standing at the point $(1,1,f(1,1))$ and walk towards the origin, will you be moving up or down on the surface? (2 points)

Up

Problem X. Suppose the surface given by the function $z = f(x,y)$ has $f(3,2) = 11$ $f_x(3,2) = 4$, and $f_y(3,2) = 5$. Please answer the following: (10 points total)

(a) Find the equation of the tangent plane to the surface when $(x,y) = (3,2)$. (6 points)

$$\begin{vmatrix} i & j & k \\ 1 & 0 & 4 \\ 0 & 1 & 5 \end{vmatrix} = \langle -4, -5, 1 \rangle$$

$$0 = -4(x-3) - 5(y-2) + (z-11)$$

$$4x + 5y - 11 = z$$

(b) Use the tangent plane found in part (a) to approximate $f(2.97, 2.02)$. (4 points)

$$f(2.97, 2.02) \approx 4(2.97) + 5(2.02) - 11$$

$$= 10.98$$

Problem XI. Given the two vectors $\mathbf{v} = \langle 2, -1 \rangle$ and $\mathbf{u} = \langle 2, 1 \rangle$ find the projection of $\mathbf{u}$ onto $\mathbf{v}$ and illustrate both $\mathbf{u}$, $\mathbf{v}$, and the projection of $\mathbf{u}$ onto $\mathbf{v}$ on the axes provided. (5 points)

$$\text{Proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \cdot \vec{v}$$

$$\frac{3}{5} \langle 2, -1 \rangle$$

$$= \left\langle \frac{6}{5}, \frac{-3}{5} \right\rangle$$