Pledge:

3/31/2010 MATH361 Calculus III Name: Solution
Dr. Lunsford Quiz 7 (30 Points Total)

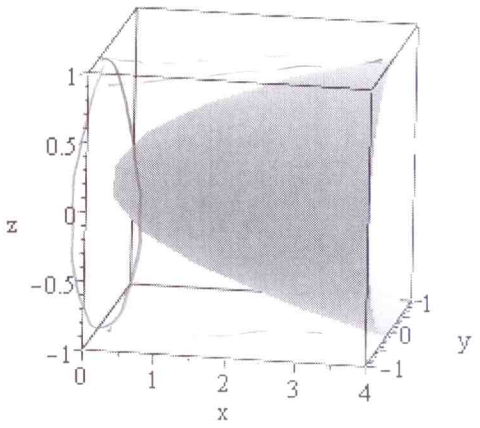
**Please show all work on this quiz including ANY substitutions you may make.**

**Problem I**. Find the limits of integration to find the integral $\iiint_R x\,dV$ on the solid region bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$. (5 points each, 10 points total)

(a) Using rectangular coordinates:

$$\iint_D \int_{4y^2+4z^2}^{4} x\,dx\,dA$$

$$= \int_{-1}^{1} \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_{4y^2+4z^2}^{4} x\,dx\,dy\,dz$$




(b) Using an appropriate polar substitution:

$$\int_0^{2\pi} \int_0^{1} \int_{4r^2}^{4} x\,dx\,r\,dr\,d\theta$$

**Problem II.** Use cylindrical coordinates to find $\iiint_E \sqrt{x^2 + y^2}\,dV$ where $E$ is the solid region bounded by the first quadrant, the cylinder $x^2 + y^2 = 9$ and the plane $z = 4$. (10 points)

$$\int_0^{4} \int_0^{\pi/2} \int_0^{3} r\,r\,dr\,d\theta\,dz$$

$$= \int_0^{4} \int_0^{\pi/2} \int_0^{3} r^2\,dr\,d\theta\,dz = 18\pi$$

**Problem III.** If $\mathbf{F}$ is the vector field given by $\mathbf{F}(x,y) = x^2y^3\,\mathbf{i} - y\sqrt{x}\,\mathbf{j}$ and $C$ is the curve given by $\mathbf{r}(t) = t^2\,\mathbf{i} - t^3\,\mathbf{j}$ for $0 \le t \le 1$ then find the line integral $\int_C \mathbf{F}\bullet d\mathbf{r}$.

$$\int_0^1 \vec{F}(x,y)\cdot\vec{r}\,'(t)\,dt \qquad \vec{r}\,'(t) = \langle 2t, -3t^2\rangle$$

$$\downarrow$$

$$\int_0^1 \langle (t^2)^2(-t^3)^3, -(-t^3)\sqrt{t^2}\rangle \cdot \langle 2t, -3t^2\rangle\,dt$$

$$= \int_0^1 -2t^{14} - 3t^6\,dt$$

$$= \frac{-59}{105}$$