Pledge:

3/24/2010 MATH361 Calculus III Name: Solution
Dr. Lunsford Quiz 5 (40 Points Total)

Please show all work on this quiz.

$$\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin(2u) + C$$

$$\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin(2u) + C$$

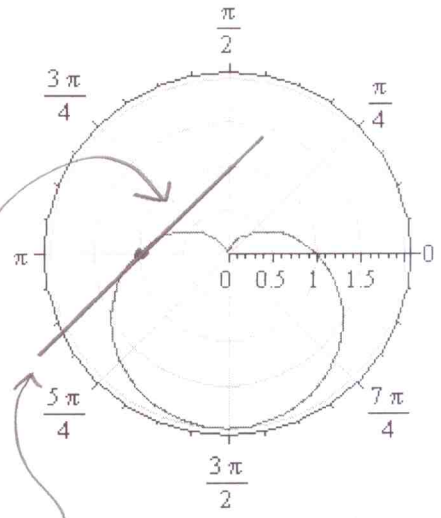
Problem I. Below you are given the polar graph of $r(\theta) = 1 - \sin(\theta)$. Find the slope of the tangent line to r at $\theta = \pi$. Draw the tangent line on the graph. (12 points)

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r'(\theta)\sin\theta + r(\theta)\cos\theta}{r'(\theta)\cos\theta - r(\theta)\sin\theta}$$

$r(\pi) = 1$

$r'(\theta) = -\cos(\theta)$, $r'(\pi) = 1$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi} = \frac{1(0) + (1)(-1)}{(1)(-1) - (1)(0)} = \frac{-1}{-1} = \boxed{1}$$



notice the tangent line is parallel to $\theta = \frac{\pi}{4}$

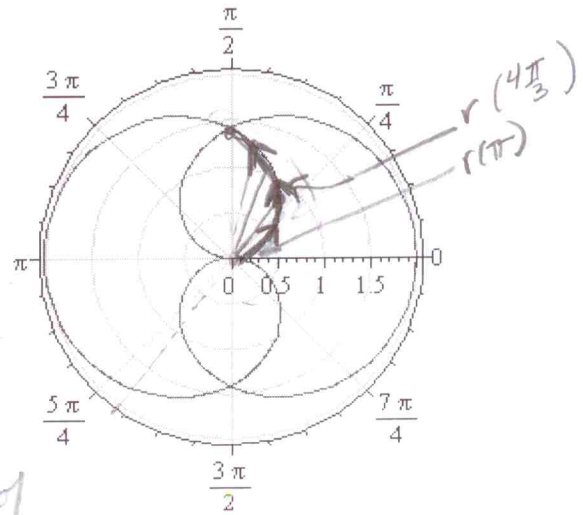
Problem II. Below you are given the polar graph of $r(\theta) = 2\cos(\theta/2)$. Indicate the portion of the graph that is drawn when $\pi \le \theta \le \frac{3\pi}{2}$. (5 points)

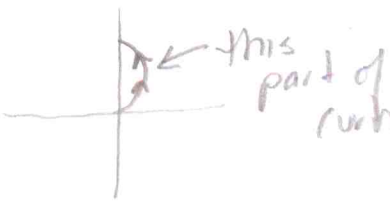
$r(\pi) = 2\cos\left(\frac{\pi}{2}\right) = 0$

$r\left(\frac{4\pi}{3}\right) = 2\cos\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{2}}{3}\cdot 2$

$r\left(\frac{5\pi}{4}\right) = 2\cos\left(\frac{5\pi}{8}\right) < 0$

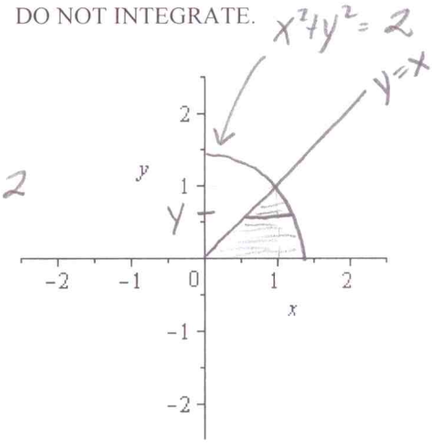
$r\left(\frac{3\pi}{2}\right) = 2\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}\cdot 2$





Problem III. Convert the integral $\int_0^1 \int_y^{\sqrt{2-y^2}} (x+y)\,dx\,dy$ to polar coordinates. DO NOT INTEGRATE.

Use the axes provided to your right to graph the region over which you are integrating. (10 points)



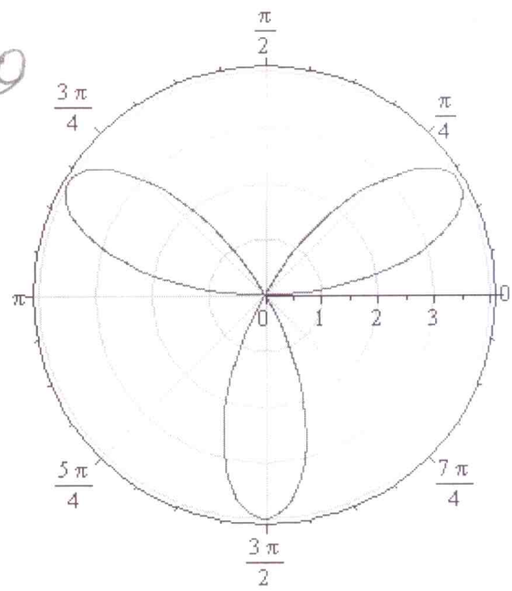
$0 \le y \le 1$

$y \le x \le \sqrt{2-y^2}$ $\rightarrow$ $x = \sqrt{2-y^2}$

$x^2 = 2-y^2 \Rightarrow x^2+y^2 = 2$

$$\int_0^{\pi/4} \int_0^{\sqrt{2}} (r\cos\theta + r\sin\theta)\, r\, dr\, d\theta$$

Problem IV. Below you are given the polar graph of $r(\theta) = 4\sin(3\theta)$. Find the area enclosed by the graph. Hint: The graph is completely drawn for some $\theta < 2\pi$. Neatly show all work. (13 points)



$$\int_0^{\pi} \frac{(r(\theta))^2}{2}\, d\theta \quad \text{OR} \quad 3\int_0^{\pi/3} \frac{(r(\theta))^2}{2}\, d\theta$$

$$= \int_0^{\pi} 8\sin^2(3\theta)\, d\theta$$

$(u = 3\theta,\ du = 3\,d\theta)$

$$= \int_0^{3\pi} \frac{8}{3}\sin^2 u\, du$$

$$= \frac{8}{3}\left[\frac{1}{2}u - \frac{1}{4}\sin(2u)\right]_0^{3\pi} = \frac{8}{3}\cdot\frac{3\pi}{2} = \boxed{4\pi}$$