Pledge:

3/15/2010 MATH361 Calculus III Name: Solution
Dr. Lunsford Quiz 4 (40 Points Total)

Please show all work on this quiz.

Problem I. Let $f(x,y) = x\sin y$. Please answer the following questions. (10 points total)

$|\langle 1,1\rangle| = \sqrt{2}$

(a) Find the directional derivative at the point $\left(3, \frac{\pi}{4}\right)$ in the direction of the vector $\langle 1,1\rangle$. (6 points)

$f_x(x,y) = \sin y$

$f_y(x,y) = x\cos y$

$\nabla f(3, \frac{\pi}{4}) = \langle \sin\frac{\pi}{4}, 3\cos\frac{\pi}{4} \rangle$

$= \langle \frac{\sqrt{2}}{2}, \frac{3\sqrt{2}}{2} \rangle$

$\nabla f \cdot \frac{\vec{u}}{|\vec{u}|} = \langle \frac{\sqrt{2}}{2}, \frac{3\sqrt{2}}{2} \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$= \frac{1}{2} + \frac{3}{2} = 2$

(b) Find the direction of the maximum rate of change of f and the value of that rate of change. Verify the value you find is greater than the value of the derivative in part (a). (4 points)

Direction: $\langle \frac{\sqrt{2}}{2}, \frac{3\sqrt{2}}{2} \rangle$

Value: $|\langle \frac{\sqrt{2}}{2}, \frac{3\sqrt{2}}{2} \rangle|$

$= \sqrt{\frac{2}{4} + \frac{18}{4}} = \sqrt{5}$

$\sqrt{5} > 2$

Problem II. Let $f(x,y) = e^{4xy - x^2 - y^3}$. Find all critical points of f. (5 points)

$f_x(x,y) = (4y - 2x)e^{4xy - x^2 - y^3}$

$f_y(x,y) = (4x - 3y^2)e^{4xy - x^2 - y^3}$

$\left.\begin{array}{l} f_x(x,y) = 0 \\ f_y(x,y) = 0 \end{array}\right\} \Rightarrow \begin{array}{l} 4y - 2x = 0 \Rightarrow y = \frac{1}{2}x \\ 4x - 3(\frac{1}{2}x)^2 = 0 \end{array}$

$x(4 - \frac{3}{4}x) = 0$

$x = 0, \quad x = \frac{16}{3}$

$y = 0 \quad y = \frac{8}{3}$

$\boxed{(0,0) \text{ and } (\frac{16}{3}, \frac{8}{3})}$

Problem III. You would like to find the point on the plane $x - y + z = 4$ that is closest to the point $(1,2,3)$. Determine a function of two variables that has a minimum or maximum value at the desired point on the plane. Indicate whether you want to maximize or minimize the function. DO NOT find the point on the plane. (5 points)

$P = (x-1)^2 + (y-2)^2 + (z-3)^2$

$P(x,y) = (x-1)^2 + (y-2)^2 + (4 - x + y - 3)^2$

minimize $P(x,y)$.

Problem IV. Let $f(x,y)=x^2-2xy+\frac{1}{3}y^3-3y$. The critical points of this function are $(3,3)$ and $(-1,-1)$. For each point determine whether it gives a local maximum, a local minimum, or a saddle point for f. (10 points)

$f_x = 2x-2y$

$f_y = -2x+y^2-3$

$f_{xx} = 2$

$f_{yy} = 2y$

$f_{xy} = -2$

$D(x,y) = f_{xx}f_{yy}-(f_{xy})^2 = 4y-4$

@ $(3,3)$: $D(3,3)>0$, $f_{xx}>0$ $\therefore$ $(3,3)$ is a local min

@ $(-1,-1)$ $D(-1,-1)<0$ $\therefore$ $(-1,-1)$ is a saddle node.

Problem V. Evaluate the iterated integral $\int_0^2\int_{x^3/2}^{2x} x+y\,dy\,dx$. (5 points)

$$\int_0^2 \left[xy+\tfrac{1}{2}y^2\right]_{x^3/2}^{2x} dx = \int_0^2 4x^2-\frac{x^4}{2}-\frac{x^6}{16}\,dx$$

$$= \frac{4}{3}x^3-\frac{x^5}{10}-\frac{x^7}{7(16)}\Big|_0^2 = \frac{554}{105}$$

Problem VI. Reverse the order of integration for the integral in Problem V. To your right you are given an unlabelled graph that you may want to use (caution – this graph may not have the same scales on the two axes). DO NOT integrate the new integral. (5 points)

$$\int_0^4\int_{y/2}^{\sqrt[3]{2y}} x+y\;dx\,dy$$