Pledge:

/15/2010
r. Lunsford

MATH361 Calculus III
Quiz 3

Name: Solution
(50 Points Total)

Please show all work on this quiz.

Problem I. Given the space curve $\mathbf{r}(t) = \sin(2t)\mathbf{i} + \cos(2t)\mathbf{j} + \sqrt{5}\, t\mathbf{k}$ please answer the following: (18 points total)

(a) Find the unit tangent vector, $\mathbf{T}(t)$, to the curve. (6 points)

$\vec{T}(t) = \dfrac{\vec{r}'(t)}{|\vec{r}'(t)|} = \langle \frac{2}{3}\cos 2t, -\frac{2}{3}\sin 2t, \frac{\sqrt{5}}{3} \rangle$

$\vec{r}'(t) = \langle 2\cos 2t, -2\sin 2t, \sqrt{5} \rangle$

$|\vec{r}'(t)| = \sqrt{4\cos^2 2t + 4\sin^2 2t + 5} = \sqrt{4+5} = 3$

(b) Find $\mathbf{T}(\pi)$. (2 points)

$\vec{T}(\pi) = \langle \frac{2}{3}, 0, \frac{\sqrt{5}}{3} \rangle$

(c) Find the unit normal vector, $\mathbf{N}(t)$, to the curve. (5 points)

$\vec{N}(t) = \dfrac{\vec{T}'(t)}{|\vec{T}'(t)|}$, $\vec{T}'(t) = \langle -\frac{4}{3}\sin 2t, -\frac{4}{3}\cos 2t, 0 \rangle$

$\Rightarrow \vec{N}(t) = \langle -\sin 2t, -\cos 2t, 0 \rangle$

$|\vec{T}'(t)| = \sqrt{(\frac{4}{3})^2 \sin^2 2t + (\frac{4}{3})^2 \cos^2 2t} = \frac{4}{3}$

(d) Find $\mathbf{N}(\pi)$. (2 points)

$N(\pi) = \langle 0, -1, 0 \rangle$

(e) Find the binormal vector $\mathbf{B}(\pi)$. (3 points)

$$\begin{vmatrix} i & j & k \\ \frac{2}{3} & 0 & \frac{\sqrt{5}}{3} \\ 0 & -1 & 0 \end{vmatrix} = \langle \frac{5}{3}, 0, \frac{-2}{3} \rangle$$

II. Suppose the space curve has the following unit tangent and unit normal vectors at the point $(-3,4,5)$:

$\mathbf{T} = \left\langle \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2} \right\rangle$, $\mathbf{N} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0 \right\rangle$. Please answer the following: (8 points total)

(a) Find the equation of the normal plane to the curve at the point. (4 points)

$\frac{1}{2}(x+3) + \frac{1}{2}(y-4) + \frac{\sqrt{2}}{2}(z-5) = 0$

(b) Find the equation of the osculating plane to the curve at the point. (4 points)

$$\vec{B} = \begin{vmatrix} i & j & k \\ \frac{1}{2} & \frac{1}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{vmatrix} = \langle -\frac{1}{2}, \frac{1}{2}, 0 \rangle$$

$\boxed{-\frac{1}{2}(x+3) + \frac{1}{2}(y-4) = 0}$

$\overset{\swarrow}{0}\ v(0)$

III. A tennis ball is projected from ground level with initial velocity $\mathbf{v}(t) = 10\mathbf{i} + 19.6\mathbf{j}$ meters per second. Recall the gravity constant is $\mathbf{a}(t) = -9.8\mathbf{j}$ meters per second squared. Below you are given a graph of the position of the ball. Please answer the following: (12 points total)

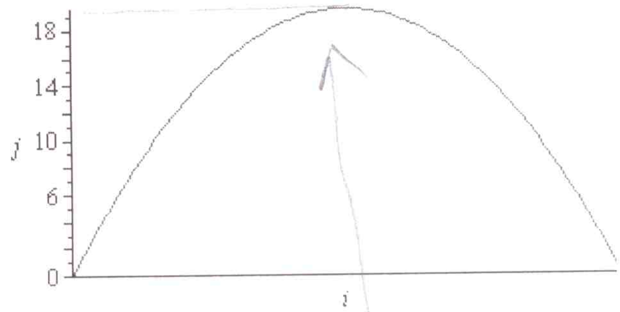
(a) Find the vector equation of motion of the ball. (6 points)

$\vec{a}(t) = \langle 0, -9.8 \rangle$

$\vec{v}(0) = \langle 10, 19.6 \rangle$

$\vec{r}(0) = \langle 0, 0 \rangle$

$\vec{r}(t) = \langle 10t, -\frac{9.8t^2}{2} + 19.6t \rangle$



(b) At what time does the ball hit the ground? (2 points)

$-\frac{9.8}{2}t^2 + 19.6t = 0$

$t(-\frac{9.8}{2}t + 19.6) = 0 \rightarrow t = 4 \text{ sec.}$

(c) How far does the ball travel horizontally? (2 points)

$10t\Big|_{t=4} = 40 m.$

(d) What is the maximum height reached by the ball? (2 points)

$\vec{v}(t) = \langle 10, -9.8t + 19.6 \rangle$

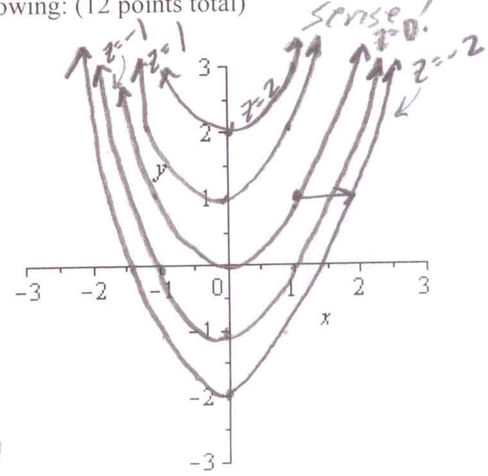
$-9.8t + 19.6 = 0 \Rightarrow t = 2 \rightarrow$ @ $t = 2$:

$-\frac{9.8}{2}(2)^2 + 19.6(2) = 19.6m$

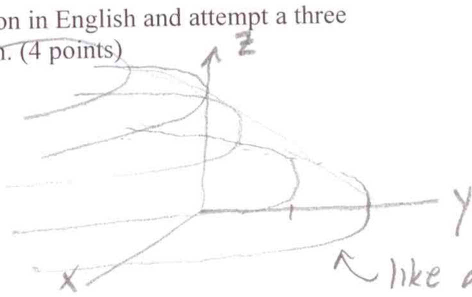
makes sense!

IV. Given the multivariable function $f(x, y) = y - x^2$ please answer the following: (12 points total)

(a) Graph the level curves for the levels $-2, -1, 0, 1, 2$. Clearly label the curves. (6 points)



(b) Describe the graph of the function in English and attempt a three dimensional drawing of the function. (4 points)



like a ship

(c) If you are on the surface of this function at the point $(1, 1, f(1,1))$ and walk in the positive x direction, will you be moving up or down the surface? (2 points)

down - level curves decreasing.