Pledge:

1/2/2010 MATH361 Calculus III Name: Solution
Dr. Lunsford Quiz 2 (30 Points Total)

Please show all work on this quiz.

Problem I. Given the points $P(2,1,5)$, $Q(-1,3,4)$, and $R(3,0,6)$ please answer the following: (8 points total)

(a) Find a unit vector orthogonal (i.e. perpendicular) to the plane through the three points. (5 points)

$\vec{PQ} = \langle -3, 2, -1 \rangle$ $\quad \vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 2 & -1 \\ 1 & -1 & 1 \end{vmatrix} = \langle 1, 2, 1 \rangle$

unit vector $= \frac{1}{\sqrt{6}} \langle 1, 2, 1 \rangle$

(b) Find the area of the triangle ΔPQR. (3 points)

$$\frac{|u \times v|}{2} = \frac{\sqrt{6}}{2}$$

Problem II. Consider the plane given by the equation $2x - 3y + 4z = 4$ and the line given by the parametric equations $x = 3 + 3t$, $y = 2 - 2t$, and $z = 1 - 3t$. Please answer the following: (11 points total)

(a) Give a vector perpendicular to the plane. (2 points) $\langle 2, -3, 4 \rangle$ (from equation)

(b) Is the point $P(3,2,1)$ on the plane? Why or why not? (3 points)

$2(3) - 3(2) + 4(1) = 4$ Yes, the point satisfies the equation of the plane

(c) Find a vector parallel to the line. (2 points)

$\langle 3, -2, -3 \rangle$ (from parametric equations)

(d) Does the line lie on the plane? Why or why not? (4 points)

I $2(3+3t) - 3(2-2t) + 4(1-3t)$
$= 6 + 6t - 6 + 6t + 4 - 12t$
$= 4$ so the parametric equations satisfy the equation of the plane

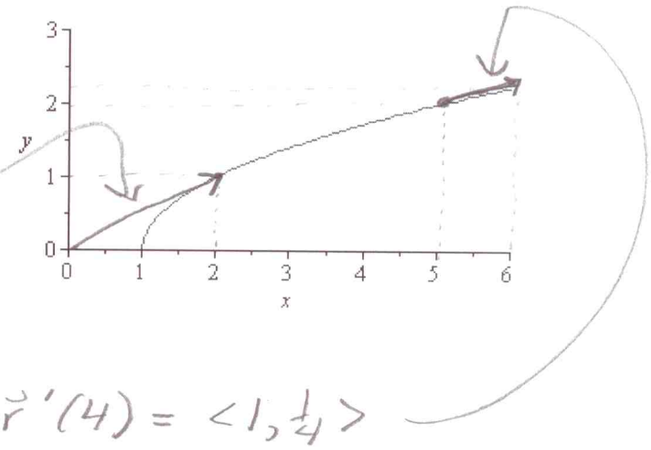
II. The point on the line $P(3,2,1)$ is on the plane.
Also $\langle 3, -2, -3 \rangle \cdot \langle 2, -3, 4 \rangle = 0$
so the direction of the line is $\perp$ to the orthog. vector of the plane.
Yes, the line is on the plane.

Problem III. Consider a point that moves along the plane curve given by $\mathbf{r}(t) = (1+t)\,\mathbf{i} + \sqrt{t}\,\mathbf{j}$. To your right you are given a graph of this curve.
Please answer the following: (6 points total)

(a) Sketch the position vector $\mathbf{r}(1)$ on the graph. (2 points)

$\vec{r}(1) = 2\vec{i} + \vec{j}$

(b) Sketch the tangent (velocity) vector to the graph at $t = 4$. (4 points)



$\vec{r}(4) = 5\vec{i} + 2\vec{j}$

$\vec{r}'(t) = \vec{i} + \frac{1}{2\sqrt{t}}\vec{j} \Rightarrow \vec{r}'(4) = \langle 1, \frac{1}{4} \rangle$

Problem IV. Consider the space curve given by $\mathbf{r}(t) = \cos(3t)\,\mathbf{i} + e^{3t}\,\mathbf{j} + 2\sin(2t)\,\mathbf{k}$. Find the unit tangent vector to the curve at $t = 0$. (5 points)

$\vec{r}'(t) = \langle -3\sin(3t),\ 3e^{3t},\ 4\cos(2t) \rangle$

$\vec{r}'(0) = \langle 0, 3, 4 \rangle$

$|\vec{r}'(0)| = 5$

tangent unit vector $\vec{T}(0) = \dfrac{\vec{r}'(0)}{|\vec{r}'(0)|} = \langle 0, \frac{3}{5}, \frac{4}{5} \rangle$