Pledge:

1/25/2010 MATH361 Calculus III Name: Solution
Dr. Lunsford Quiz 1 (40 Points Total)

Please show all work on this quiz.

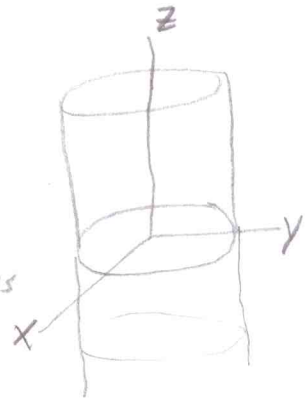
Problem I. Describe (in English) the following graphs in $\mathbf{R}^3$: (3 points each, 6 total)

(a) $x = 3$

A plane parallel to the yz plane and through the point (3,0,0)

(b) $x^2 + y^2 = 25$

A vertical cylinder through the circle w/ center (0,0,0) and radius 5 on the xy plane

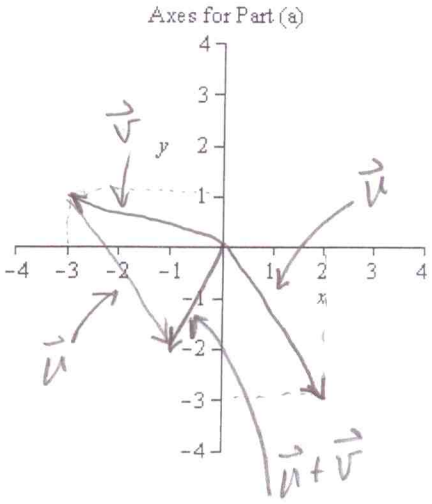


Problem II. Given the vectors $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$ and $\mathbf{v} = -3\mathbf{i} + \mathbf{j}$ in $\mathbf{R}^2$ find each of the following and illustrate on the axes provided: (14 points total)

(a) $\mathbf{u} + \mathbf{v}$ (5 points)

$= \langle -1, -2 \rangle = -\vec{i} - 2\vec{j}$

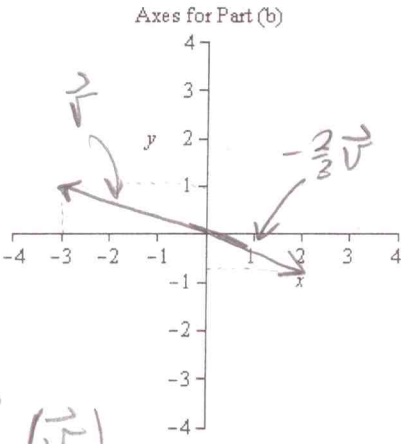
Axes for Part (a)



(b) $-\frac{2}{3}\mathbf{v}$ (4 points)

$= -\frac{2}{3}\langle -3, 1 \rangle = \langle 2, -\frac{2}{3} \rangle$

Axes for Part (b)

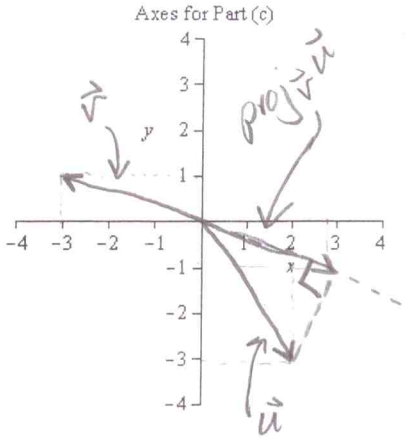


(c) $proj_{\mathbf{v}}\mathbf{u}$ (5 points)

$= \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|}\left(\frac{\vec{v}}{|\vec{v}|}\right) = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}(\vec{v})$

$= \frac{-9}{10}\langle -3, 1 \rangle = \langle \frac{27}{10}, -\frac{9}{10} \rangle$

Axes for Part (c)



Problem III. What is the angle (in degrees) between the vectors $\mathbf{u}$ and $\mathbf{v}$ given in Problem II above? (4 points)

$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \frac{-9}{\sqrt{130}} \Rightarrow \theta = \arccos\left(-\frac{9}{130}\right)$

$\theta = 142.125°$

Problem IV. Now consider the vectors **u** and **v** given in Problem II as vectors in $\mathbf{R}^3$ (i.e. they are both on the *xy* plane). Please answer the following: (6 points total)

(a) Find the magnitude of $\mathbf{u} \times \mathbf{v}$. (3 points)

$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin\theta = |\sqrt{13}||\sqrt{10}|\sin(142.125°) = 7$

Note: computationally (not required here) we get:

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 0 \\ -3 & 1 & 0 \end{vmatrix} = \langle 0, 0, -7 \rangle$$

(b) In which direction does $\mathbf{u} \times \mathbf{v}$ point? (3 points)

Via the right hand rule,
in the negative z direction $(-\vec{k})$

Problem V. Please write "true" or "false," according to which is correct, in the blank provided next to each statement. (2 points each, 10 total)

False The dot product is a vector in $\mathbf{R}^2$.

False The cross product is defined in $\mathbf{R}^2$ and $\mathbf{R}^3$.

True The cross product $\mathbf{u} \times \mathbf{v}$ is a vector that is perpendicular to both **u** and **v**.

True $\mathbf{k} \times \mathbf{j} = -\mathbf{i}$

False $\mathbf{k} \bullet \mathbf{j} = 1$ $(\vec{k} \cdot \vec{j} = 1)$