Pledge:

4/21/2010 MATH361 Calculus III Name:_______________________

Dr. Lunsford Test 2 (100 Points Total)

Please show all work on this test. You may (or may not) find the following formulas useful.

$$\int \sin^2 u\, du = \frac{1}{2}u - \frac{1}{4}\sin(2u) \qquad \int \cos^2 u\, du = \frac{1}{2}u + \frac{1}{4}\sin(2u) \qquad \iint_S 1 dS = \iint_D |\mathbf{r}_u \times \mathbf{r}_v| dA$$

$$\int f(x,y,z)ds = \int_a^b f(\mathbf{r}(t)) |\mathbf{r}'(t)| dt \qquad \int \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$

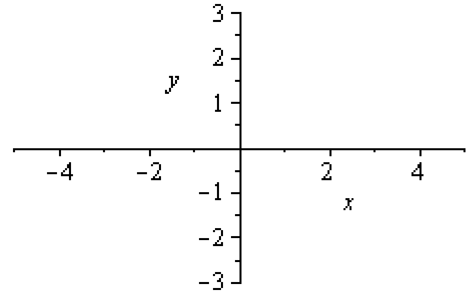
$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F} \cdot \mathbf{T} ds = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\, dt = \int_C P\, dx + Q\, dy \qquad \oint_C F \cdot dr = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

Problem I. Find the maximum rate of change of $f(x,y) = \frac{x^2}{y}$ at the point $(4,2)$ and the direction in which it occurs. Clearly indicate your answers. (8 points)

Problem II. Find the directional derivative of $f(x,y) = \ln(x^2 + y^2)$ at the point $(2,1)$ in the direction of the vector $\mathbf{v} = \langle -1, 2 \rangle$. (8 points)

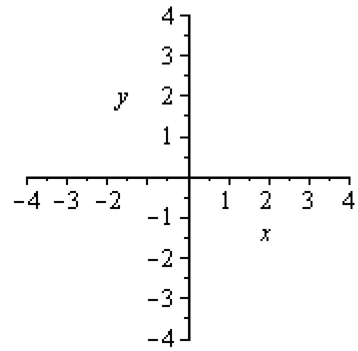
Problem III. Change the order of integration for the following integral. On the axes provided include a graph of the region *R* over which the integral is defined. DO NOT EVALUATE the integral. (10 points)

$$\int_0^4 \int_{\sqrt{x}}^2 \cos(y^2)\, dy dx$$



Problem IV. Set up, but DO NOT EVALUATE, an integral to find the surface area of the parametric surface given by the vector function $\mathbf{r}(u,v) = v^2\mathbf{i} - uv\mathbf{j} + u^2\mathbf{k}$ for $0 \le u \le 3$ and $-3 \le v \le 3$. (10 points)

Problem V. Change the following integral to polar coordinates: $\iint_R (x+y)\,dA$ where R is the region below the x-axis and between the graphs of $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$. On the axes provided include a graph of the region R over which the integral is defined. DO NOT EVALUATE the integral. (10 points)



Problem VI. Evaluate the line integral $\int_C y\sin(z)\,ds$ where C is the circular helix given by $x = \cos(t)$, $y = \sin(t)$ and $z = t$, $0 \le t \le 2\pi$. (10 points)

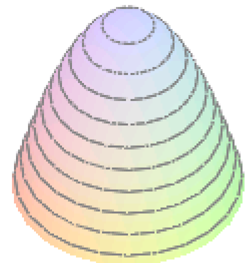
<u>Problem VII.</u> Use <u>Green's Theorem</u> to evaluate $\int_C x^4 dx + xydy$ where C is the triangular curve, oriented counterclockwise, consisting of the line segments from (0,0) to (2,0), from (2,0) to (0,1), and from (0,1) to (0,0). (10 points)

<u>Problem VIII.</u> Let $\mathbf{F} = (3y - 2z)\mathbf{i} + (3x + z)\mathbf{j} + (y - 2x)\mathbf{k}$. Please answer the following questions: (16 points total)

(a) Find $f(x, y, z)$ such that $\mathbf{F} = \nabla f$. (10 points)

(b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the line segment from (1,2,3) to (1,4,5). (6 points)

Problem IX. We wish to evaluate the integral $\iiint_B yz\,dV$ where B is the solid region below the paraboloid $z = 18 - 2x^2 - 2y^2$ and above the xy-plane. To your right you are given a graph of the solid with the lines on the solid drawn parallel to the xy-plane. Please answer the following: (12 points total)



(a) Set up, DO NOT EVALUATE, the integral in rectangular coordinates. (6 points)

(b) Set up, DO NOT EVALUATE, the integral in cylindrical coordinates. (6 points)

Problem X. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle xy^2, x^2y \rangle$ and C is the path, oriented counterclockwise, consisting of the line segment from (0,0) to (1,0), then along the circular arc $y = \sqrt{1-x^2}$ from (1,0) to (0,1), then along the line segment from (1,0) to (0,0). The graph below may be helpful. Hint: Think conservative! (6 points)