Pledge:

2/24/2010 MATH361 Calculus III Name:________________________
Dr. Lunsford Test 1 (100 Points Total)

Please show all work on this test.

Problem I. Find the angle (in degrees) between the two vectors $\mathbf{v} = \langle 2,2,1 \rangle$ and $\mathbf{u} = \langle 1,2,2 \rangle$. (5 points)

Problem II. Consider the points $A = (-2,1,0)$, $B = (3,3,-1)$, and $C = (0,2,0)$. Please answer the following: (14 points total)

(a) Find the equation of the plane through the three points. (10 points)

(b) Find the area of the triangle enclosed by the three points. (4 points)

Problem III. Consider the two lines given by the following parametric equations:

Line 1: $x = 3 - t$, $y = 2 + t$, $z = 1 - t$
Line 2: $x = 3 + 2s$, $y = -2 - s$, $z = 5 + s$

Determine if the two lines are parallel, intersecting, or skew. (10 points)

Problem IV. Let $\mathbf{u} = \langle 3, 4x + y + 1, 4 \rangle$ and $\mathbf{v} = \langle x + y, 2, y \rangle$. Find values of x and y such that $\mathbf{u}$ and $\mathbf{v}$ are orthogonal. Note that there is more than one possible answer here! Check to make sure the two values of x and y you find are correct. (10 points)

Problem V. Suppose the velocity vector for an object moving in space is $\mathbf{v}(t) = \langle t^2, e^{2t}, \sin(t) \rangle$ where t is in seconds and $\mathbf{v}(t)$ is in feet per second for each component. Please answer the following: (10 points total)

(a) Find the position vector, $\mathbf{r}(t)$, for the object if $\mathbf{r}(0) = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$. (6 points)

(b) Find the acceleration vector for the object at time $t = 0$. (4 points)

Problem VI. Consider the surface $z = f(x, y) = x \ln(x^2 - y^3) + 3xy$. Please answer the following: (12 points total)

(a) Find $f_x(x, y)$ and $f_y(x, y)$. DO NOT simplify your answers! (8 points)

(b) Find the slope of the tangent line to the curve of intersection of the surface and the plane $x = 3$, at the point $(3, 2, 36)$. (4 points)

Problem VII. If $z = x^2 + xy^3$, $x = uv^2 + w^3$ and $y = u + ve^w$ use the Chain Rule to find $\dfrac{\partial z}{\partial u}$ when $u = 2$, $v = 1$, and $w = 0$. (8 points)

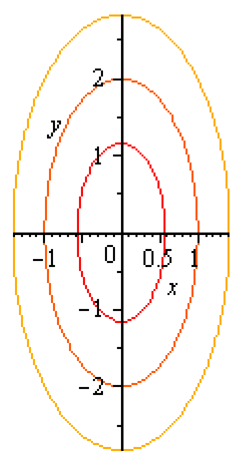
Problem VIII. For each limit below determine if the limit exists or not. Justify your answers. (5 points each – 10 total)

(a) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+2y^2}$

(b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{3x^3}{x^2+2y^2}$

Problem IX. To your right you are given the level curves for the function $f(x,y) = -x^2 - \dfrac{y^2}{4}$ for the levels $-\dfrac{1}{3}$, -1 and -2. Please answer the following: (6 points total)

(a) Clearly label which curve is for each of the three levels given (i.e. match the curve to its level). (4 points)



(b) If you are standing at the point $(1,1, f(1,1))$ and walk towards the origin, will you be moving up or down on the surface? (2 points)

Problem X. Suppose the surface given by the function $z = f(x, y)$ has $f(3,2) = 11$ $f_x(3,2) = 4$, and $f_y(3,2) = 5$. Please answer the following: (10 points total)

(a) Find the equation of the tangent plane to the surface when $(x, y) = (3,2)$. (6 points)

(b) Use the tangent plane found in part (a) to approximate $f(2.97, 2.02)$. (4 points)

Problem XI. Given the two vectors $\mathbf{v} = \langle 2, -1 \rangle$ and $\mathbf{u} = \langle 2, 1 \rangle$ find the projection of $\mathbf{u}$ onto $\mathbf{v}$ and illustrate both $\mathbf{u}$, $\mathbf{v}$, and the projection of $\mathbf{u}$ onto $\mathbf{v}$ on the axes provided. (5 points)