

Pledge:

2/20/2007
Dr. Lunsford

MATH 271
Quiz 3

Name: Solution
20 Points Possible

To assess the accuracy of a laboratory scale, a standard weight known to weigh 10 grams is weighed repeatedly. The scale readings are normally distributed with *unknown* mean μ . Note that $\mu = 10g$ if the scale is unbiased. The standard deviation of the scale readings on this scale is known to be 0.0002 grams. Please answer the following. Please be sure to show all calculator input for full credit. $\rightarrow \sigma$

(a) The weight is weighed five times. The mean result is 10.0023 grams. Find a 98% confidence interval for μ , the true mean of the scale readings. Write a complete English sentence explaining the result of your confidence interval. (4 points)

Z-interval: $\sigma = .0002$
 $\bar{X} = 10.0023$

We are 98% confident that the true mean of the scale readings for the 10g weight is between 10.002g and 10.003g.

$(10.002, 10.003)$

If use t interval: $(10.001, 10.002)$

(b) How many measurements must be averaged to get a margin of error of ± 0.0001 with 98% confidence? (4 points)

$\bar{X} + m$, $m = \frac{z^* \sigma}{\sqrt{n}} \Rightarrow n = \left(\frac{z^* \sigma}{m} \right)^2 = \left(\frac{2.366(0.0002)}{.0001} \right)^2 = 22.33$
m.o.e.

For 98% confidence $z^* = \text{invNorm}(.99) = 2.363$

$\boxed{n=23}$

(c) A lab technician believes the scale has a positive bias meaning it is reporting higher weights than it should be. Based on the confidence interval you found in part (a), ~~what~~ do you think there is evidence to support the technician's belief? Why or why not? (2 points)

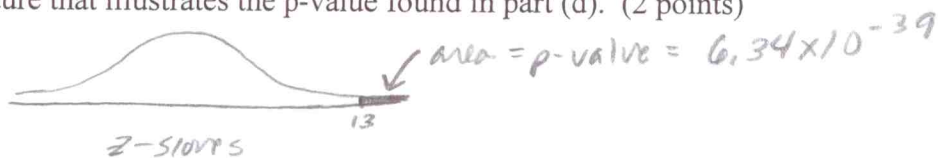
Yes, there is evidence to support the belief. Since the entire C.I. is above 10. Thus the scale tends to overweigh the 10g weight, on average.

(d) If you conduct the following hypothesis test below for the true mean reading of the scale when weighing a 10g weight, then what are the values of the test statistic and the p-value for the test? Assume the same sample mean weight for 4 weightings was 10.0013. (4 points) Test: $H_0: \mu = 10$ versus $H_a: \mu > 10$

$z = 13$

$p = 6.34 \times 10^{-39}$ ($= 4.91 \times 10^{-4}$ if use t-test)

(e) Draw a picture that illustrates the p-value found in part (d). (2 points)



(f) What is your conclusion of the test conducted in part (d) in terms of the hypotheses and in the context of this problem? (4 points)

Reject H_0 in favor of H_1 since p-value is very small.
Thus the true mean of the scale readings for the 10g weight is above 10g.