Pledge:

9/20/2011 MATH261 Calculus I Name: Solution
Dr. Lunsford Quiz 4 (40 Points Total)

Find the exact value of the following limits (i.e. not a calculator approximation) provided the limit exists. If the limit does not exist as a number, then determine if it exists in the infinite sense (i.e. equals plus or minus infinity). For each limit you must neatly show at least one intermediate step for full credit. (5 points each)

1. $\lim_{t\to\pi} \sin\left(\pi - \frac{7t}{2}\right)$ $= \sin\left(\pi - \frac{7\pi}{2}\right) = \sin\left(-\frac{5\pi}{2}\right) = \boxed{-1}$


2. $\lim_{x\to 0}\left[\frac{2\cos x}{1-\cos^2 x} - \frac{1}{1-\cos x}\right]$ $= \lim_{x\to 0}\left[\frac{2\cos x}{(1-\cos x)(1+\cos x)} - \frac{1(1+\cos x)}{(1-\cos x)(1+\cos x)}\right]$

$= \lim_{x\to 0}\left[\frac{2\cos x - 1 - \cos x}{(1-\cos x)(1+\cos x)}\right] = \lim_{x\to 0} \frac{\cos x - 1}{(1-\cos x)(1+\cos x)}$

$= \lim_{x\to 0} \frac{-1}{1+\cos x} = \frac{-1}{1+1} = \boxed{-\frac{1}{2}}$

3. $\lim_{\theta\to -\frac{\pi}{4}} \frac{\sin(3\theta)}{\theta}$ $= \frac{\sin\left(-\frac{3\pi}{4}\right)}{-\frac{\pi}{4}} = -\frac{\sqrt{2}}{2} \cdot -\frac{4}{\pi} = \boxed{\frac{2\sqrt{2}}{\pi}}$

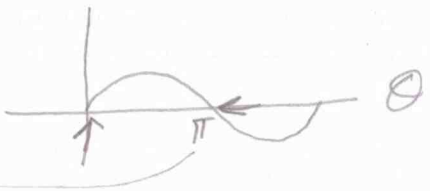

4. $\lim_{h\to 0^-} \frac{\sin(4h)}{|h|}$ $= \lim_{h\to 0^-} \frac{\sin(4h)}{-h} = \lim_{h\to 0^-} \left[-1\left(\frac{\sin(4h)}{h}\right)\right]$

↳ 0/0 indet.

$|h| = \begin{cases} h, & h \geq 0 \\ -h, & h < 0 \end{cases}$ $= -1 \cdot 4 = \boxed{-4}$

5. $\lim_{\theta \to \pi^+} \frac{\theta^2 - 9}{\sin\theta} = \boxed{-\infty}$

$\hookrightarrow \frac{\pi^2 - 9}{0^-}$ ← positive constant




6. $\lim_{w \to 0} \frac{\tan^2 w}{1 - \sec w} = \lim_{w \to 0} \frac{\sec^2 w - 1}{1 - \sec w} = \lim_{w \to 0} \frac{(\sec w - 1)(\sec w + 1)}{1 - \sec w}$

$\hookrightarrow \frac{0}{0}$

$\sin^2 w + \cos^2 w = 1$

$\Rightarrow \tan^2 w + 1 = \sec^2 w$

$= \lim_{w \to 0} -(\sec w + 1) = \boxed{-2}$

7. $\lim_{\theta \to 0^+} \frac{\cos\theta - 2}{\theta} = \boxed{-\infty}$

$\hookrightarrow \frac{-1}{0^+}$

8. $\lim_{\theta \to 0^+} \frac{\tan(4\theta)}{3\theta} = \lim_{\theta \to 0^+} \frac{\frac{\sin(4\theta)}{\cos(4\theta)}}{3\theta}$

$= \lim_{\theta \to 0^+} \frac{1}{3} \cdot \frac{\sin(4\theta)}{\theta} \cdot \frac{1}{\cos(4\theta)} = \boxed{\frac{4}{3}}$

$\downarrow \quad \downarrow$

$\frac{1}{3} \cdot 4 \cdot 1$