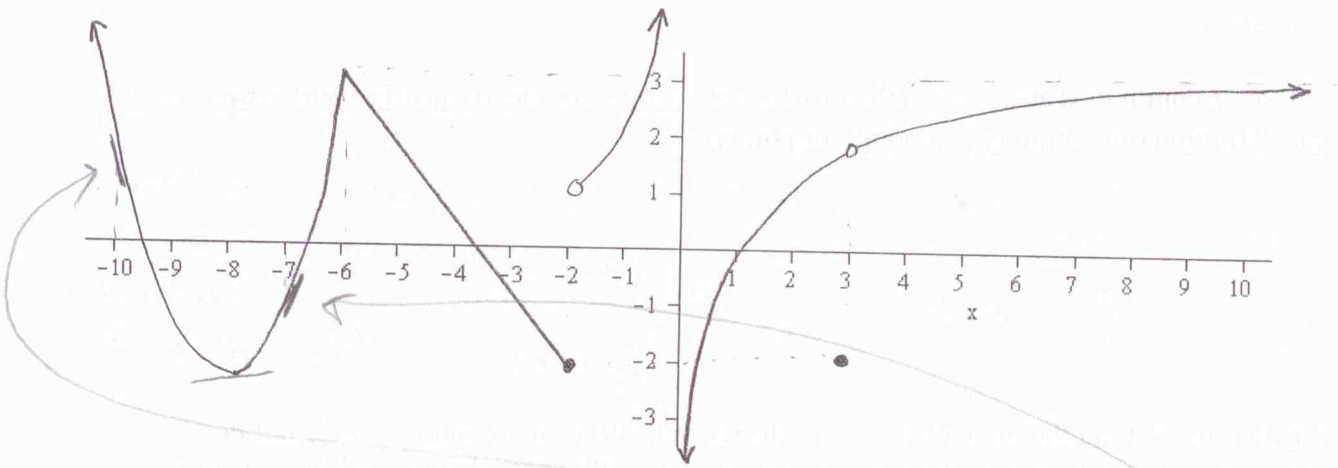
10/3/2011
Dr. Lunsford

MATH 261 Calculus I
Test 1

Name: Solution
(100 Points Total)

For all limits on this test, determine if the limit exists as a number, exists in the infinite sense, or does not exist. If the limit exists find its value.

Problem I. Use the graph of the function f below to answer the following questions. (2 points each – 32 total)



$f(3) =$ -2 $\quad \lim_{x\to 3} f(x) =$ 2 $\quad \lim_{x\to -2^+} f(x) =$ 1 $\quad \lim_{x\to -2} f(x) =$ DNE

$\lim_{x\to\infty} f(x) =$ 3 $\quad \lim_{x\to -6} f(x) =$ 3 $\quad \lim_{x\to 1^-} \frac{x}{f(x)} =$ $-\infty$ $\quad \lim_{x\to 0^+} f(x) =$ $-\infty$

$\frac{1}{0^-}$

For the remaining questions, please write "true" or "false", according to which is correct about the statement, in the space provided next to each statement.

false f is continuous at $x = 3$.

True f is continuous from the left at $x = -2$.

True f is continuous on the interval $[-\infty, -2]$.

Cont from left at -2

True $f'(-10) < f'(-7)$.

negative positive

List all x values at which f has a removable discontinuity. $x = 3$

List all x values at which f is continuous but not differentiable. $x = -6$

List all x values at which $f'(x) = 0$. $x = -8$

Write down the equation of any vertical asymptote(s) that f may have:

$x = 0$ since $\lim_{x\to 0^+} f(x) = -\infty$

$\lim_{x\to 0^-} f(x) = \infty$

Problem II. Use the function $f(x)=\begin{cases} x+1, & x\le -2 \\ x^2+1, & -2<x<2 \\ 3+x, & 2\le x\end{cases}$ to answer the following questions.

For full credit you must show at least one intermediate step for any limits you compute. (6 points total)

(a) $\lim\limits_{x\to -1^+} f(x) = \lim\limits_{x\to -1^+} (x^2+1) = 2$

(2 points)

(b) Is f continuous at $x=2$? Why or why not? <u>You must clearly justify your answer using the definition of continuity at a point</u>. (4 points)

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} 3+x = 5$

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} x^2+1 = 5$

So, $\lim\limits_{x\to 2} f(x) = 5$

and $f(2) = 3+2 = 5$

since $\lim\limits_{x\to 2} f(x) = 5 = f(2)$ → then f is continuous at $x=2$

Problem III. Find the indicated limits – clearly indicate your answers. *You do not need to show any intermediate steps for these problems*. <u>Very little partial credit will be given for these problems</u>. (3 points each, 18 total)

1. $\lim\limits_{x\to -1} 7x^{13}+9x^{14}-18x^{15} = 20$

2. $\lim\limits_{x\to 1^+} \sqrt{1-x^2} = $ DNE   (∞ $x\to 1^+$ $1-x^2\to 0^-$)

3. $\lim\limits_{x\to 3^-} \dfrac{1-x}{3-x} = -\infty$ ($\dfrac{-2}{0^+}$)

4. $\lim\limits_{x\to -\infty} 7x^{13}+9x^{14}\boxed{-18x^{15}} = \infty$

5. $\lim\limits_{x\to \pi/4} \dfrac{\cos(2x)}{x} = 0$ ($\cos\left(\frac{\pi}{2}\right) = \dfrac{0}{\pi/4}$, $\dfrac{\pi}{4}$)

6. $\lim\limits_{x\to\infty} \dfrac{7\boxed{-3x^4}}{\boxed{6x^4}+11x^2+13} = -\dfrac{1}{2}$

Problem IV. Use the <u>limit definition of the derivative function</u> to show that if $f(x)=\sqrt{2x+1}$ then $f'(x)=\dfrac{1}{\sqrt{2x+1}}$. (8 points)

$$\lim_{t\to x}\frac{f(t)-f(x)}{t-x} = \lim_{t\to x}\frac{\sqrt{2t+1}-\sqrt{2x+1}}{t-x}$$

$$= \lim_{t\to x}\frac{2t+1-(2x+1)}{(t-x)(\sqrt{2t+1}+\sqrt{2x+1})}$$

$$= \lim_{t\to x}\frac{2(\cancel{t-x})}{(\cancel{t-x})(\sqrt{2t+1}+\sqrt{2x+1})}$$

$$= \lim_{t\to x}\frac{2}{(\sqrt{2t+1}+\sqrt{2x+1})} = \frac{2}{2\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}$$

$$= \lim_{h\to 0}\frac{(2x+2h+1)-(2x+1)}{h(\sqrt{2x+2h+1}+\sqrt{2x+1})}$$

$$= \lim_{h\to 0}\frac{2h}{h(\sqrt{2x+2h+1}+\sqrt{2x+1})}$$

$$= \lim_{h\to 0}\frac{2}{\sqrt{2x+2h+1}+\sqrt{2x+1}}$$

$$= \frac{2}{2\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$

Problem V. Find the indicated limits. You must show at least one intermediate step to receive full credit. (5 points each – 25 points total)

1. $\lim_{x\to 1^+} \frac{1-x^2}{x^2+7x-8}$ $\Rightarrow$ $\lim_{x\to 1^+} \frac{(1-x)(1+x)}{(x-1)(x+8)} = \lim_{x\to 1^+} \frac{-(1+x)}{x+8}$

$\hookrightarrow \frac{0}{0}$

Indet Form

$= \boxed{-\frac{2}{9}}$

2. $\lim_{\theta\to 0^+} \frac{\sin(3\theta)}{\sqrt{\theta}} \cdot \frac{\sqrt{\theta}}{\sqrt{\theta}} = \lim_{\theta\to 0^+} \left(\frac{\sin(3\theta)}{\theta}\right)\left(\sqrt{\theta}\right) = 0$

$\hookrightarrow \frac{0}{0}$

Indet. Form.

$\downarrow \; 3 \cdot 0$

3. $\lim_{u\to 0} u^2 \sin\left(\frac{1}{u^2}\right) = 0$

Hint: What is a nice theorem to use if you are between a rock and a hard place?

$-1 \le \sin\left(\frac{1}{u^2}\right) \le 1$ $\quad -u^2 \le u^2 \sin\left(\frac{1}{u^2}\right) \le u^2$

$u^2 > 0$ as $u \to 0$

so

as $u \to 0$ $\quad \downarrow 0$ $\quad$ so by squeezing theorem $\downarrow 0$ $\quad \downarrow 0$

4. $\lim_{x\to -\infty} \frac{\sqrt{x^6-9}}{2x^3-6}$ $\longrightarrow$ behaves like $\sqrt{x^6}$

$\longrightarrow$ behaves like $2x^3$

$\frac{\infty}{-\infty}$

$\hookrightarrow$ So if a limit exists it will be a negative number

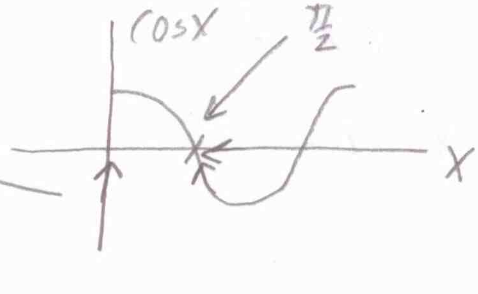
$= \lim_{x\to -\infty} \frac{\sqrt{x^6}}{2x^3} = \lim_{x\to -\infty} \frac{|x|^3}{2x^3} = \lim_{x\to -\infty} \frac{(-x)^3}{2x^3}$ since $x \to -\infty$

$= -\frac{1}{2}$

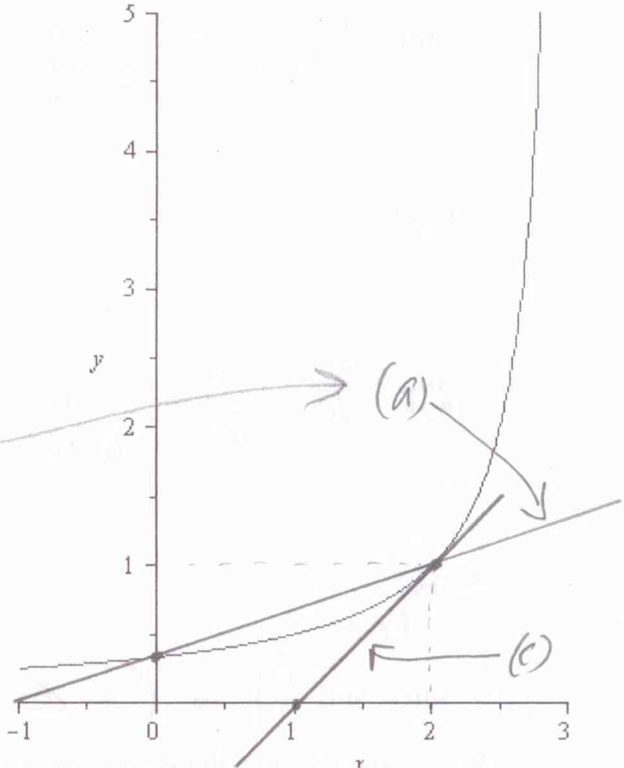
5. $\lim_{x\to \frac{\pi}{2}^+} \frac{x+\frac{\pi}{2}}{\cos x} = -\infty$

$\hookrightarrow \frac{\pi}{\cos\frac{\pi}{2}} = \frac{\pi}{0^-}$

plug



Problem VI. Below you are given part of the graph of the function $f(x) = \dfrac{1}{3-x}$. Please answer the following questions: (11 points total)



(a) Find the slope of the secant line through $x = 0$ and $x = 2$. Draw this line on the same axes as the function. (3 points)

$$\frac{\Delta f}{\Delta x} = \frac{f(2)-f(0)}{2-0} = \frac{1-\frac{1}{3}}{2-0}$$

$$= \frac{\frac{2}{3}}{2} = \frac{1}{3}$$

(b) Find the slope of the tangent line to f at $x = 2$. **<u>You must use the limit definition for any credit</u>**. (5 points)

$$\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2} \frac{\frac{1}{3-x} - 1}{x-2}$$

$$= \lim_{x\to 2} \frac{1-(3-x)}{(x-2)(3-x)} = \lim_{x\to 2} \frac{x-2}{(x-2)(3-x)} = \lim_{x\to 2} \frac{1}{3-x} = 1$$

OR $$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{\frac{1}{3-(2+h)} - 1}{h} = \lim_{h\to 0} \frac{\frac{1}{1-h} - 1}{h}$$

$$= \lim_{h\to 0} \frac{1-(1-h)}{h(1-h)} = \lim_{h\to 0} \frac{h}{h(1-h)} = \lim_{h\to 0} \frac{1}{1-h} = 1$$

(c) Find the equation of the tangent line to f at $x = 2$ and accurately graph it on the same axes as the function. (3 points)

slope (from part (b)) is 1

Point on line $(2, f(2)) = (2, 1)$

$$y - 1 = 1(x-2)$$

$$y = x - 1$$