Pledge:

11/7/2011 MATH261 Calculus I Name: Solution
Dr. Lunsford Quiz 9 (40 Points Total)

You must neatly show all work on this quiz. You may (or may not) find the following formulas useful:

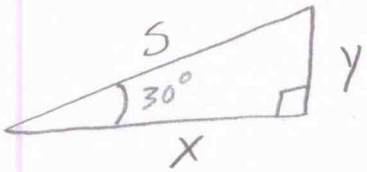
$$V=\frac{4}{3}\pi r^3,\ V=\frac{1}{3}\pi r^2 h,\ S=4\pi r^2,\ A=\pi r^2,\ C=2\pi r$$

1. A mountain climber hikes up a 30 degree slope at a rate of ½ foot per second.

(a) How fast is the climber's vertical distance changing? (8 points)

Given $\frac{ds}{dt}=\frac{1}{2}$ ft/s

Want to find $\frac{dy}{dt}$



$\sin 30^\circ=\frac{y}{s} \Rightarrow \sin 30^\circ s = y$

$\frac{d}{dt}\sin 30^\circ s=\frac{d}{dt}y \Rightarrow \sin 30^\circ \frac{ds}{dt}=\frac{dy}{dt} \Rightarrow \frac{dy}{dt}=\frac{1}{2}\left(\frac{1}{2}\text{ ft/s}\right)$

$=\frac{1}{4}$ ft/s

(b) How fast is the climber's horizontal distance changing? (8 points)

Find $\frac{dx}{dt}$

$\cos 30^\circ=\frac{x}{s} \Rightarrow \cos 30^\circ s = x$

$\frac{d}{dt}\cos 30^\circ s=\frac{d}{dt}x \Rightarrow \cos 30^\circ \frac{ds}{dt}=\frac{dx}{dt}$

$\frac{dx}{dt}=\frac{\sqrt{3}}{2}\left(\frac{1}{2}\text{ ft/s}\right)=\frac{\sqrt{3}}{4}$ ft/s

2. Air is being pumped into a spherical balloon at the rate of 2π cubic inches per minute. How fast is the radius of the balloon changing when its surface area is 16π square inches? (9 points)

$V=\frac{4}{3}\pi r^3$ ← spherical balloon → $\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$

$\frac{dV}{dt}=2\pi$ in^3/s

$2\pi=4\pi(4)\frac{dr}{dt}$

Find $\left.\frac{dr}{dt}\right|_{A=16\pi}$

$\frac{dr}{dt}=\frac{1}{8}$ in/min

Note: when $A=16\pi \Rightarrow S=4\pi r^2$

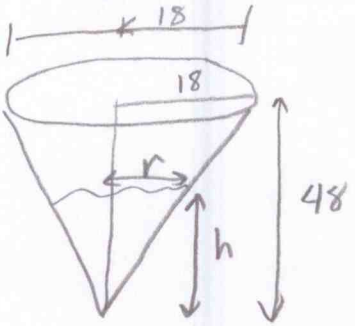
$16\pi=4\pi r^2$

$r=2$

Pledge:

3. Water is flowing into an inverted canonical tank at the rate of 30 cubic inches per minute. The width of the tank is 36 inches and its height is 48 inches. Unfortunately, water is also leaking out of the tip at the bottom of the tank. The rate of the leak depends on how much water is in the tank, specifically, the rate of the leak is $2h$ cubic inches per minute where h is the height of the water in the tank.

(a) How fast is the height of the water changing when the height is 10 inches? Is the water level (i.e. height of the water in the tank) rising or falling at that moment? (10 points)



Given $\frac{dV}{dt}_{in} = 30 - 2h$

$V = \frac{1}{3}\pi r^2 h$

$\frac{r}{h} = \frac{18}{48}$

$r = \frac{18}{48}h = \frac{3}{8}h$

$V = \frac{1}{3}\pi\left(\frac{3}{8}h\right)^2 h$

$V = \frac{3}{64}\pi h^3$

$\frac{dV}{dt} = \frac{9\pi}{64}h^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{64}{9\pi h^2}\frac{dV}{dt}$

$\left.\frac{dh}{dt}\right|_{h=10} = \frac{64}{(9\pi(100))}(30-20) = \frac{32}{45\pi}$ in/min

(b) Is there a height at which the water level is neither rising nor falling? If so, what is that height? (5 points)

$\frac{dh}{dt} = \frac{64}{9\pi h^2}(30-2h)$

$\frac{dh}{dt} = 0 \iff 30-2h = 0$

$\iff h = 15$ in.