Pledge:

9/27/2011
Dr. Lunsford

MATH261 Calculus I
Quiz 5

Name: Solution
(40 Points Total)

I. Find the exact value of the following limits (i.e. not a calculator approximation) provided the limit exists. If the limit does not exist as a number, then determine if it exists in the infinite sense (i.e. equals plus or minus infinity). **You do not need to show any work for these problems.** (3 points each – 6 total)

1. $\lim_{x\to -3^+} \frac{\overset{-2}{1+x}}{\underset{0^+}{x+3}} = \boxed{-\infty}$

$-3 \quad -2$

2. $\lim_{x\to -\infty} \frac{3\boxed{-x^5}}{x+2x^3\boxed{+4x^5}+11x^4} = \boxed{-\frac{1}{4}}$

II. Find the exact value of the following limits (i.e. not a calculator approximation) provided the limit exists. If the limit does not exist as a number, then determine if it exists in the infinite sense (i.e. equals plus or minus infinity). For each limit **you must neatly show at least one intermediate step for full credit.** (5 points each, 20 total)

1. $\lim_{t\to 2^-} \frac{t^2-4}{|t-2|} = \lim_{t\to 2^-} \frac{(t-2)(t+2)}{-(t-2)} = \lim_{t\to 2^-} -(t+2) = \boxed{-4}$

$\frac{0}{0}$ Indet.

$|t-2| = \begin{cases} t-2, & t-2 \geq 0 \text{ or } t \geq 2 \\ -(t-2), & t-2<0 \text{ or } t<2 \end{cases}$

2. $\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3 \cancel{-x^3}}{h}$

Yes, this limit is written correctly.
There are no typos.

$\frac{(x+0)^3-x^3}{0} = \frac{0}{0}$ Indet!

$= \lim_{h\to 0} \frac{h(3x^2+3xh+h^2)}{h}$

$= \lim_{h\to 0} 3x^2+3xh+h^2 = \boxed{3x^2}$

3. $\lim_{x\to -\infty} \frac{3x^3-2x^2+1000}{\sqrt{16x^6-3x^4+25}}$

behaves like $3x^3$ as $x\to\infty$

behaves like $\sqrt{16x^6} = 4|x|^3$ $= 4(-x)^3$

behaves like limit $\lim_{x\to -\infty} \frac{3x^3}{4|x|^3} = \lim_{x\to -\infty} \frac{3x^3}{-4x^3}$

$= -\frac{3}{4}$

plug: $\frac{-\infty}{\infty}$ Indet

say answer will be negative

II. Finding limits showing your work, continued

4. $\displaystyle\lim_{h\to-\infty}\frac{\sin(4h)}{h} = \boxed{0}$

Hint: What is that cute little theorem you love to hug? Using the squeezing theorem.

$-1 \le \sin(4h) \le 1$ for all h

$$-\frac{1}{h} \ge \underbrace{\frac{\sin(4h)}{h}}_{S.T.\ \downarrow\ 0} \ge \frac{1}{h} \quad \text{since } h<0$$

$$\downarrow \qquad\qquad \downarrow \quad \text{as } h\to-\infty$$

$$0 \qquad\qquad 0$$

III. Use the function $f(x)=\dfrac{2x^2-3x-2}{x^2-3x+2}$ to answer the following questions. (14 points total)

(a) What is the domain of f ? (2 points) $(-\infty,1)\cup(1,2)\cup(2,\infty)$

$x^2-3x+2=0 \Rightarrow (x-2)(x-1)=0 \Rightarrow x=1$ or $x=2$

(b) Find *the equations* of all horizontal asymptotes of f. You must justify your answers by taking the appropriate limits. Clearly indicate your answers. (6 points)

$\displaystyle\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{2x^2-3x-2}{x^2-3x+2} = 2$ → only one h.a. @ $\boxed{y=2}$

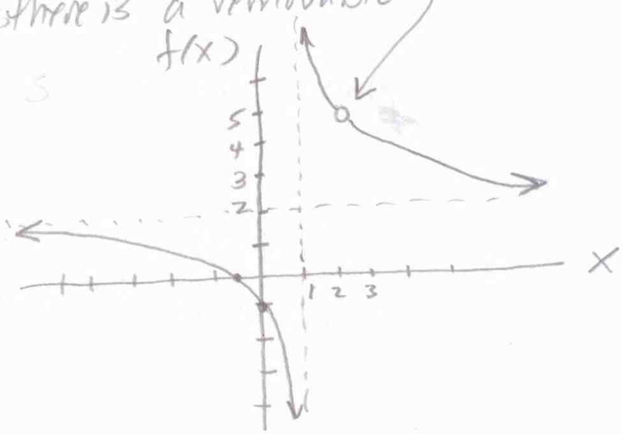
$\displaystyle\lim_{x\to-\infty} f(x) = 2$

(c) Find *the equations* of all vertical asymptotes of f. You must justify your answers by taking the appropriate limits. Clearly indicate your answers. (6 points)

$\displaystyle\lim_{x\to2} f(x) = \lim_{x\to2}\frac{(2x+1)(x-2)}{(x-1)(x-2)} = \lim_{x\to2}\frac{2x+1}{x-1} = 5$

↳ indet $\frac{0}{0}$

so $x=2$ is not a v.a. (note there is a removable discontinuity @ $x=2$).



$\displaystyle\lim_{x\to1^+} f(x) = \lim_{x\to1^+}\frac{\overset{3}{2x+1}}{\underset{0^+}{x-1}} = \infty$

so $\boxed{x=1 \text{ is a v.a.}}$