

Pledge:

11/30/2011
Dr. Lunsford

MATH261 Calculus I
Quiz 11

Name: Solution
(50 Points Total)

You must neatly show all work on this quiz.

I. Find the indicated integrals. Explicitly show any substitutions you may make. You must show all work to receive any credit. (6 points each, 30 total)

1. $\int x^3 - 2\sqrt[4]{x^3} + 3 dx = \int x^3 - 2x^{3/4} + 3 dx$

$$= \frac{1}{4}x^4 - 2\frac{4}{7}x^{7/4} + 3x + C = \boxed{\frac{1}{4}x^4 - \frac{8}{7}x^{7/4} + 3x + C}$$

2. $\int \frac{t^3 - \sqrt{t} - 2}{t} dt$

$$= \int t^2 - t^{-1/2} - 2t^{-1} dt = \boxed{\frac{1}{3}t^3 + 2t^{1/2} - 2\ln|t| + C}$$

3. $\int xe^{x^2} dx = \frac{1}{2} \int \underbrace{2x}_{u'} e^{\overbrace{x^2}^u} dx = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C$
 $u = x^2$
 $du = 2x dx$
 $= \boxed{\frac{1}{2} e^{x^2} + C}$

4. $\int \cos^4(3\theta) \sin(3\theta) d\theta = \int (\cos(3\theta))^4 (\sin(3\theta)) d\theta$

$$u = \cos(3\theta)$$

$$du = -3\sin(3\theta) d\theta$$

$$-\frac{1}{3} du = \sin(3\theta) d\theta$$

$$\int u^4 \left(-\frac{1}{3} du\right) = -\frac{1}{3} \left(\frac{1}{5} u^5\right) + C$$

$$= \boxed{-\frac{1}{15} \cos^5(3\theta) + C}$$

5. $\int x(1-x)^8 dx$

$$u = 1-x \Rightarrow x = 1-u$$

$$du = -dx$$

$$\int (1-u)u^8 (-du) = \int u^9 - u^8 du$$

$$= \frac{1}{10} u^{10} - \frac{1}{9} u^9 + C = \boxed{\frac{1}{10} (1-x)^{10} - \frac{1}{9} (1-x)^9 + C}$$

Pledge:

II. A car traveling with velocity 24 meters per second begins to slow down at time $t = 0$ seconds with a constant deceleration of -6 meters per second squared. Please answer the following questions. (20 points total)

1. Write an initial value problem that reflects this scenario. You should include the differential equation and any initial conditions. Hint: Since you have not been given a starting position, make it something convenient. (4 points)

Let p be the position of the car at time t .

$$\begin{array}{l} \text{D.E.} \leftarrow p''(t) = -6 \rightarrow (a(t) = -6) \\ \text{I.C.} \leftarrow \begin{cases} p'(0) = 24 \rightarrow (v(0) = 24) \\ p(0) = 0 \end{cases} \end{array}$$

2. Find the velocity of the car as a function of time, i.e. find $v(t)$. (5 points)

$$v(t) = \int a(t) dt = \int (-6) dt = -6t + C$$
$$v(0) = -6(0) + C, v(0) = 24 \Rightarrow C = 24$$

$$v(t) = -6t + 24$$

3. At what time will the car come to a halt? (4 points)

$$v(t) = p'(t) = 0$$
$$\Rightarrow -6t + 24 = 0$$
$$t = 4 \text{ seconds}$$

4. What is the total distance the car travelled from when it started decelerating to when it came to a halt? (7 points)

$$p(t) = \int v(t) dt = \int -6t + 24 dt$$
$$= -3t^2 + 24t + C$$
$$p(0) = 0 \Rightarrow -3(0) + 24(0) + C = 0 \Rightarrow C = 0$$
$$p(t) = -3t^2 + 24t$$
$$p(4) - p(0) = -3(16) + 24(4) - 0$$
$$= 48 \text{ m.}$$