Pledge:

11/15/2011 MATH261 Calculus I Name: Solution

Dr. Lunsford Quiz 10 (50 Points Total)

You must neatly show all work on this quiz.

I. Find the indicated limits. If the limit does not exist as a number you should determine if it exists in the infinite sense. You must show all work to receive any credit. (5 points each, 25 total)

1. $\lim_{t\to 0}\frac{4t+7t^2}{8t^2-9t} = \lim_{t\to 0}\frac{t(4+7t)}{t(8t-9)} = \lim_{t\to 0}\frac{4+7t}{8t-9} = \boxed{-\frac{4}{9}}$ ← w/out L'Hopital

$\hookrightarrow \frac{0}{0}$ $\overset{L'H}{=} \lim_{t\to 0}\frac{4+14t}{16t-9} = \boxed{-\frac{4}{9}}$ ← w/ L'Hopital

2. $\lim_{x\to\infty}\frac{7x^3+4}{e^{3x}} \overset{L'H}{=} \lim_{x\to\infty}\frac{21x^2}{3e^{3x}} \overset{L'H}{=} \lim_{x\to\infty}\frac{42x}{9e^{3x}} \overset{L'H}{=} \lim_{x\to\infty}\frac{42}{27e^{3x}} = \boxed{0}$

$\hookrightarrow \frac{\infty}{\infty}$ $\hookrightarrow \frac{\infty}{\infty}$ $\hookrightarrow \frac{\infty}{\infty}$ $\frac{42}{\infty}$

3. $\lim_{\theta\to 0^+}\frac{\cos(\theta)}{\theta} = \boxed{+\infty}$

$\hookrightarrow \frac{1}{0^+}$

not indeterminant! Cannot use L'Hopital's Rule

4. $\lim_{w\to\infty} w\sin\left(\frac{3}{w}\right) = \lim_{w\to\infty}\frac{\sin\left(\frac{3}{w}\right)}{\frac{1}{w}} \overset{L'H}{=} \lim_{w\to\infty}\frac{\cos\left(\frac{3}{w}\right)\left(\frac{-3}{w^2}\right)}{\frac{-1}{w^2}}$

$\hookrightarrow \infty\cdot 0$ $\hookrightarrow \frac{0}{0}$

Indet. but not in form for L'Hopital's rule

$= \lim_{w\to\infty} 3\cos\left(\frac{3}{w}\right) = \boxed{3}$

5. $\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^x = y \Rightarrow \lim_{x\to\infty}\ln\left(\frac{x}{x+1}\right)^x = \ln y$

$\hookrightarrow 1^\infty$

$\Rightarrow \lim_{x\to\infty} x\ln\left(\frac{x}{x+1}\right) = \ln y \Rightarrow \lim_{x\to\infty}\frac{\ln\left(\frac{x}{x+1}\right)}{\frac{1}{x}} = \ln y$

$\hookrightarrow \infty\cdot 0$ not in form for L'Hopital's Rule

$\overset{L'H}{\Rightarrow} \lim_{x\to\infty}\frac{\frac{x+1}{x}\cdot\left(\frac{x+1-x}{(x+1)^2}\right)}{-\frac{1}{x^2}} = \ln y$

$\Rightarrow \lim_{x\to\infty}\frac{-x^2}{x(x+1)} = \ln y \Rightarrow -1 = \ln y \Rightarrow y = \boxed{\frac{1}{e}}$

Pledge: → so i.p. @ $x=0$ and $x=2$

II. Use the function $f(x) = x^4 - 4x^3$ to answer the following questions. YOU MAY NOT USE A CALCULATOR ON THIS PROBLEM. Please show all work. (25 points total)

Note: $f''(x) = 12x^2 - 24x$

$f''(x) = 0 \Rightarrow 12x(x-2) = 0$

c.t.i.p are $x=0$ and $x=2$

	$(-\infty,0)$	$(0,2)$	$(2,\infty)$
x	$-$	$+$	$+$
$x-2$	$-$	$-$	$+$
sign f''	$+$	$-$	$+$

1. What are the zeros of this function (i.e. where does it cross the x axis)? (3 points)

$\hookrightarrow f(x) = 0 \Rightarrow x^4 - 4x^3 = 0 \Rightarrow x^3(x-4) = 0 \Rightarrow \boxed{x=0 \text{ or } x=4}$

2. Find all critical points of the function. (5 points)

$f'(x) = 4x^3 - 12x^2$

$f'(x) = 0 \Rightarrow 4x^3 - 12x^2 = 0 \Rightarrow 4x^2(x-3) = 0$

$\boxed{x=0 \text{ or } x=3}$

3. Determine on which intervals the function is increasing and on which intervals the function is decreasing. It would be helpful to draw a first derivative sign chart here. Clearly indicate your answers. (5 points)

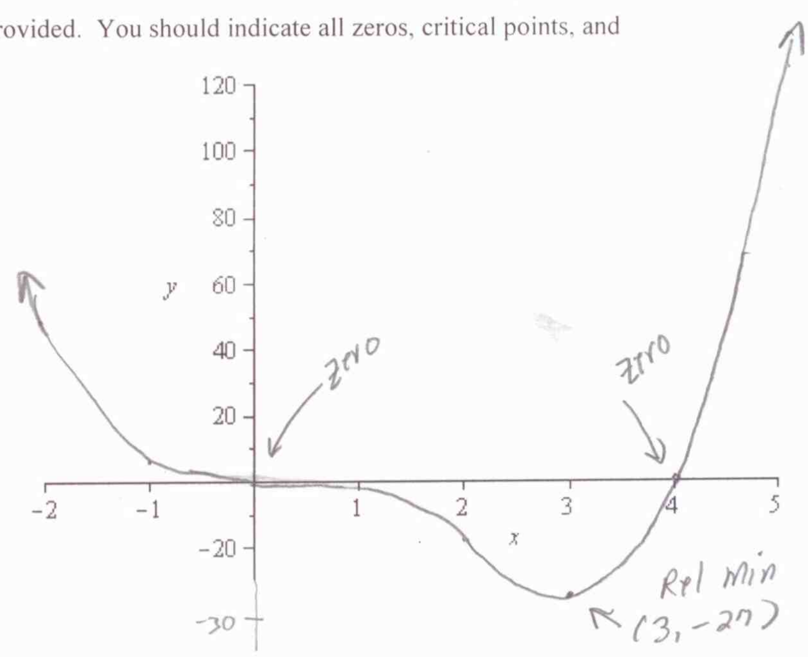
	$(-\infty,0)$	$(0,3)$	$(3,\infty)$
$4x^2$	$+$	$+$	$+$
$x-3$	$-$	$-$	$+$
sign f'	$-$	$-$	$+$

f is decreasing on $(-\infty,0)$ and $(0,3)$ (or $(-\infty,3)$) and is increasing on $(3,\infty)$

4. Find all extrema (relative maxima and relative minima) of the function. For each extrama you should indicate if it is a relative max or min and why, where the extrema occurs, and its value. (5 points)

There is a relative min. value at $x=3$ since the derivative changes from negative to positive at $x=3$. The min. value is $f(3) = 3^4 - 4(3)^3 = -27$

5. Draw a graph of the function on the axes provided. You should indicate all zeros, critical points, and extrema on the graph. (4 points)



6. Do you think any of the extrema you found above are absolute extrema? Why or why not? (3 points)

There is an absolute min at $x=3$ since f is decreasing everywhere to the left of 3 and increasing everywhere to the right of 3.