Pledge:

9/13/2011
Dr. Lunsford

MATH261 Calculus I
Quiz 3

Name: Solution
(50 Points Total)

I. Find the exact value of the following limits (i.e. not a calculator approximation) provided the limit exists. If the limit does not exist as a number, then determine if it exists in the infinite sense (i.e. equals plus or minus infinity). For each limit you must neatly show at least one intermediate step for full credit. (23 points total)

1. $\lim_{t\to 3^-} \frac{t+2}{t-3} = \boxed{-\infty}$
(4 points)

plug $\to \frac{5}{0^-}$

2. $\lim_{w\to 6^+} \sqrt{36-w^2} = \boxed{DNE}$
(4 points)

plug $\to \sqrt{0}$ how does $36-w^2$ go to zero? $36-w^2 \to 0^-$ $\therefore$ limit DNE

3. $\lim_{x\to -1^+} \frac{x^2-1}{x^3+4x^2+4x+1} = \lim_{x\to -1^+} \frac{(x-1)(x+1)}{(x+1)(x^2+3x+1)}$
(5 points)

plug $\to \frac{0}{0}$

$= \lim_{x\to -1^+} \frac{x-1}{x^2+3x+1} = \frac{-2}{-1} = \boxed{2}$

4. $\lim_{x\to -1^+} \frac{x^2-1}{x^2+2x+1} = \lim_{x\to -1^+} \frac{(x-1)(x+1)}{(x+1)(x+1)} = \lim_{x\to -1^+} \frac{x-1}{x+1} = \boxed{-\infty}$
(5 points)

plug $\to \frac{-2}{0^+}$

plug $\to \frac{0}{0}$

5. $\lim_{h\to 4^+} \left[\frac{5h}{h^2-3h-4} - \frac{4}{h-4}\right] = \lim_{h\to 4^+} \left[\frac{5h}{(h-4)(h+1)} - \frac{4}{h-4}\right]$
(5 points)

plug $\to \frac{20}{0} - \frac{4}{0}$

$= \lim_{h\to 4^+} \left[\frac{5h-4(h+1)}{(h-4)(h+1)}\right]$

$= \lim_{h\to 4^+} \frac{h-4}{(h-4)(h+1)} = \lim_{h\to 4^+} \frac{1}{h+1} = \boxed{\frac{1}{5}}$

II. Use the function below to find the indicated limits and answer the questions. (12 points total)

$$f(x) = \begin{cases} 1+x, & x \le -1 \\ x^2, & -1 < x \le 2 \\ 2x, & 2 < x \end{cases}$$

(a) $\lim_{x\to 3/2} f(x) = \lim_{x\to 3/2} x^2 = \frac{9}{4}$
(2 points)

(b) $\lim_{x\to 2.001} f(x) = \lim_{x\to 2.001} 2x = 4.002$
(2 points)

(c) $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} x^2 = 1$
(2 points)

(d) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2 = 4$
(2 points)

Is f continuous at $x = 2$? Why or why not? (4 points)

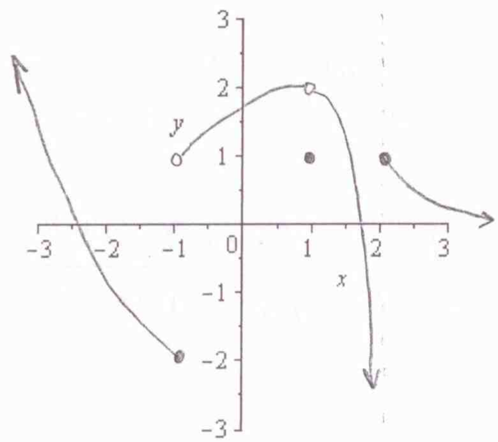
Yes b/c $\lim_{x\to 2} f(x) = f(2)$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 2x = 4$, $f(2) = 2^2 = 4$

So from above, since $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = 4$

then $\lim_{x\to 2} f(x) = 4$

III. Use the graph of the function below to answer the following questions. (3 points each, 15 points total)



At which values of x is f continuous from the right? (circle ALL that apply)

$x=-2$ $\quad$ $x=-1$ $\quad$ $x=0$ $\quad$ $x=1$ $\quad$ $x=2$ $\quad$ none of these

At which values of x is f continuous from the left? (circle ALL that apply)

$x=-2$ $\quad$ $x=-1$ $\quad$ $x=0$ $\quad$ $x=1$ $\quad$ $x=2$ $\quad$ none of these

At which values of x does f have a removable discontinuity? (circle ALL that apply)

$x=-2$ $\quad$ $x=-1$ $\quad$ $x=0$ $\quad$ $x=1$ $\quad$ $x=2$ $\quad$ none of these

At which values of x does f have a ~~step~~ jump discontinuity? (circle ALL that apply)

$x=-2$ $\quad$ $x=-1$ $\quad$ $x=0$ $\quad$ $x=1$ $\quad$ $x=2$ $\quad$ none of these

At which values of x does f have an infinite discontinuity? (circle ALL that apply)

$x=-2$ $\quad$ $x=-1$ $\quad$ $x=0$ $\quad$ $x=1$ $\quad$ $x=2$ $\quad$ none of these