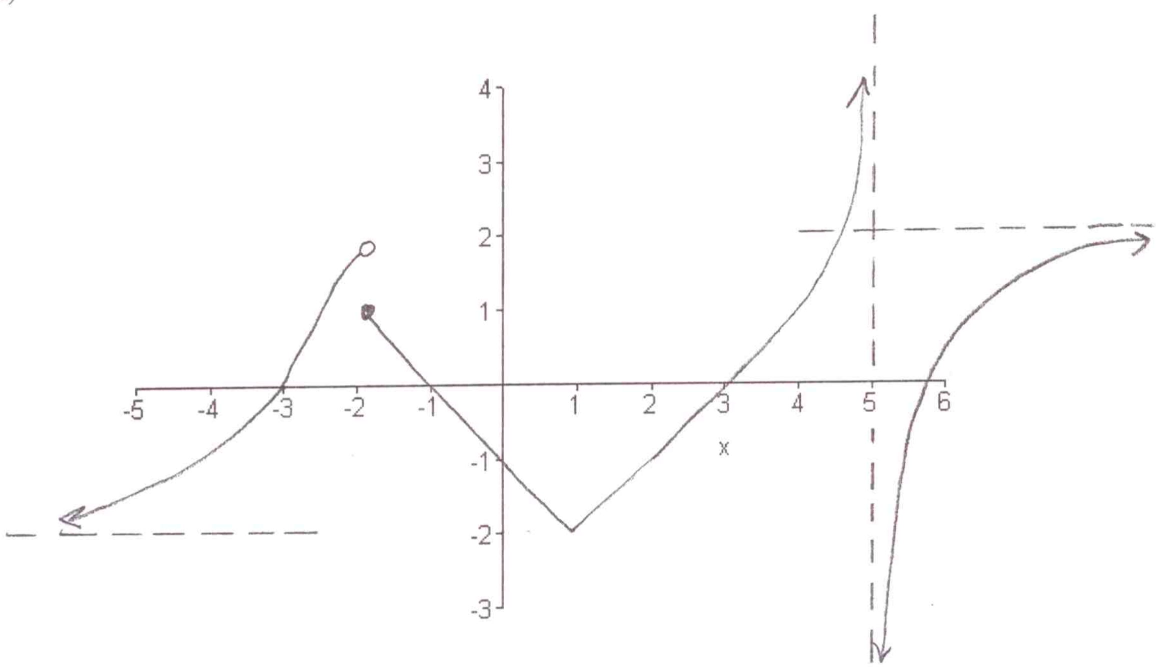
2/17/2009 MATH261 Calculus I Name: Solution

Dr. Lunsford Test 1 (100 Points Total)

For all limits on this test, determine if the limit exists as a number, exists in the infinite sense, or does not exist. If the limit exists find its value.

Problem I. Use the graph of the function f below to answer the following questions. (3 points each – 27 total)



$f(0) = \underline{-1}$ $f(3) = \underline{0}$ $\lim_{x\to -2^+} f(x) = \underline{1}$ $\lim_{x\to 5^+} f(x) = \underline{-\infty}$

$\lim_{x\to -2^-} f(x) = \underline{2}$ $\lim_{x\to -2} f(x) = \underline{DNE}$ $\lim_{x\to \infty} f(x) = \underline{2}$ $\lim_{x\to -\infty} f(x) = \underline{-2}$

$\lim_{x\to 3^-} \dfrac{x}{f(x)} = \underline{-\infty}$

For the remaining questions, please write "true" or "false", according to which is correct about the statement, in the space provided next to each statement.

<u>False</u> f is continuous at $x = 5$.

<u>True</u> f is continuous from the right at $x = -2$.

<u>True</u> f is continuous on the interval $[-2, 5)$

<u>False</u> $f'(-4) < f'(0)$

Problem II. Determine if the function given by $f(x) = \begin{cases} \dfrac{\sin x}{x}, & x < 0 \\ x^2 + 1, & 0 \le x \end{cases}$ is continuous at $x = 0$. <u>You must clearly justify your answer using the definition of continuity at a point</u>. (6 points)

$$\left.\begin{array}{l} \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 + 1 = 1 \\ \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \dfrac{\sin x}{x} = 1 \end{array}\right\} \therefore \lim_{x\to 0} f(x) = 1,\ f(0) = 1$$

$\therefore$ since $\lim_{x\to 0} f(x) = f(0)$ f is continuous at $x = 0$.

Problem III. Find the indicated limits – clearly indicate your answers. *You do not need to show any intermediate steps for these problems.* Very little partial credit will be given for these problems. (2 points each, 16 total)

1. $\lim_{x\to 2^-} \frac{x}{2-x}$ = $+\infty$ (2, 0^+)

2. $\lim_{x\to\infty} \frac{7-8x^2}{3x^2+11}$ = $-\frac{8}{3}$

3. $\lim_{x\to -1} 4x^{13} - 9x^{15} + 11x^{14}$ = $-4+9+11$ $\boxed{=16}$

4. $\lim_{x\to -\infty} 4x^{13} - 9x^{15} + 11x^{14}$ = $+\infty$

5. $\lim_{x\to -\infty} \tan^{-1}(x)$ = $-\frac{\pi}{2}$

6. $\lim_{x\to\infty} \ln\left(\frac{1}{x}\right)$ = $-\infty$ (0^+)

7. $\lim_{x\to 0^-} e^{1/x}$ = 0 ($-\infty$)

8. $\lim_{x\to 2^+} \sqrt{2-x}$ = DNE (0^-)

Problem IV. Consider the following table of values given below. Please answer the following questions (3 points each, 6 total)

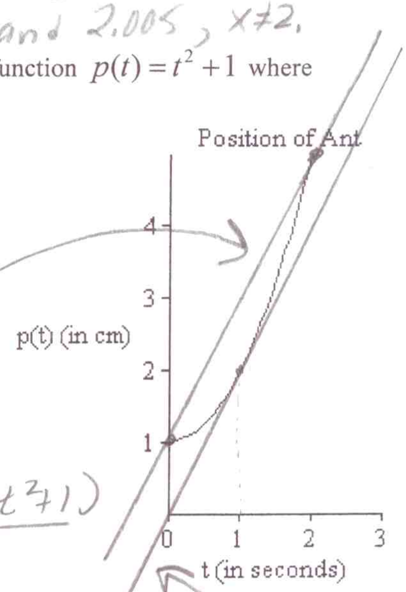
(a) From the numerical evidence, what is $\lim_{x\to 2} f(x)$?

x	1.9	1.99	1.995	2.005	2.01	2.1
$f(x)$	-4.013	-4.006	-4.002	-3.997	-3.991	-3.839

-4.00

(b) Is it possible that $\lim_{x\to 2} f(x)$ does not exist or equals some other number than your answer in part (a)? Why or why not? Yes! We can get alot closer to 2. Anything could happen to function values for x between 1.995 and 2.005, $x \neq 2$.

Problem V. An ant walks along a ruler. The ant's position is given by the function $p(t) = t^2 + 1$ where $p(t)$ is in centimeters and t is in seconds. A graph of this position function is given below. Please answer the following questions. (20 points total)



(a) Find the average velocity of the ant from time $t = 0$ to $t = 2$ seconds. Draw and clearly indicate the line on the graph whose slope represents this velocity. (5 points)

$\frac{p(2)-p(0)}{2-0} = \frac{5-1}{2} = \boxed{2 \text{ cm/s}}$

(b) Use the definition of the derivative function to show $p'(t) = 2t$. (9 points)

$p'(t) = \lim_{h\to 0} \frac{p(t+h)-p(t)}{h} = \lim_{h\to 0} \frac{(t+h)^2+1-(t^2+1)}{h}$

$= \lim_{h\to 0} \frac{t^2+2th+h^2-t^2}{h} = \lim_{h\to 0} 2t+h = 2t$

(c) Find the velocity of the ant at time $t = 1$ seconds. Find the equation of the line whose slope represents this velocity. Accurately draw and clearly indicate the line on the graph. (6 points)

$p'(1) = 2(1) = 2$ cm/s
velocity @ $t=1$

pt on line = $(1, 2)$, slope = 2
equation $y = 2(x-1)+2 = 2x$
Line: $y = 2x$

$\to 0$ $\quad \to$ DONE

Problem VI. Find the indicated limits. You must show at least one intermediate step to receive full credit. (5 points each – 25 points total)

1. $\lim_{u\to 0} u^2\cos\left(u^{-2}\right) = \lim_{u\to 0} u^2 \cos\left(\frac{1}{u^2}\right) = \boxed{0}$

Hint: What is a nice theorem to use if you are between a rock and a hard place?

$-1 \le \cos\frac{1}{u^2} \le 1$ for all $u \in \mathbb{R}$

Since $u^2 > 0$, $u \neq 0$ we have: $-u^2 \le u^2 \cos\frac{1}{u^2} \le u^2$

As $x \to 0$ both u^2 and $-u^2 \to 0$ $\therefore$ by the squeezing theorem

2. $\lim_{t\to 3} \dfrac{\left(\frac{1}{t^2+1} - \frac{1}{10}\right)}{(t-3)} \dfrac{(10)(t^2+1)}{(10)(t^2+1)} = \lim_{t\to 3} \dfrac{10-(t^2+1)}{10(t-3)(t^2+1)}$

$= \lim_{t\to 3} \dfrac{9-t^2}{10(t-3)(t^2+1)} = \lim_{t\to 3} \dfrac{-(t-3)(t+3)}{10(t-3)(t^2+1)}$

$= \lim_{t\to 3} \dfrac{-(t+3)}{10(t^2+1)} = \dfrac{-6}{100} = \boxed{\dfrac{-3}{50}}$

3. $\lim_{x\to -1^+} \dfrac{x^2+3x+2}{2x^2+x-1} = \lim_{x\to -1^+} \dfrac{(x+1)(x+2)}{(2x-1)(x+1)} = \lim_{x\to -1^+} \dfrac{x+2}{2x-1}$

$\frac{0}{0}$ Indet. Form

$= \dfrac{1}{-3} = -\dfrac{1}{3}$

IBL

4. $\lim_{x\to -\infty} \dfrac{\sqrt{x^2-9}}{2x-6} = \lim_{x\to -\infty} \dfrac{\sqrt{x^2}}{2x} = \lim_{x\to -\infty} \dfrac{|x|}{2x} = \lim_{x\to -\infty} \dfrac{-x}{2x} = -\dfrac{1}{2}$

$\frac{\infty}{\infty}$ | OR $= \lim_{x\to -\infty} \dfrac{\sqrt{x^2}\sqrt{1-9/x^2}}{2x-6} = \lim_{x\to -\infty} \dfrac{|x|\sqrt{1-9/x^2}}{2x-6}$

$= \lim_{x\to -\infty} \dfrac{-x\sqrt{1-9/x^2}}{2x-6} = \lim_{x\to -\infty} \dfrac{-\sqrt{1-9/x^2}}{2-6/x} = -\frac{1}{2}$

$-\frac{\pi}{2}$

negative

5. $\lim_{x\to -\pi^+} \dfrac{x+\frac{\pi}{2}}{\sin x} = \infty$

0^-

negative