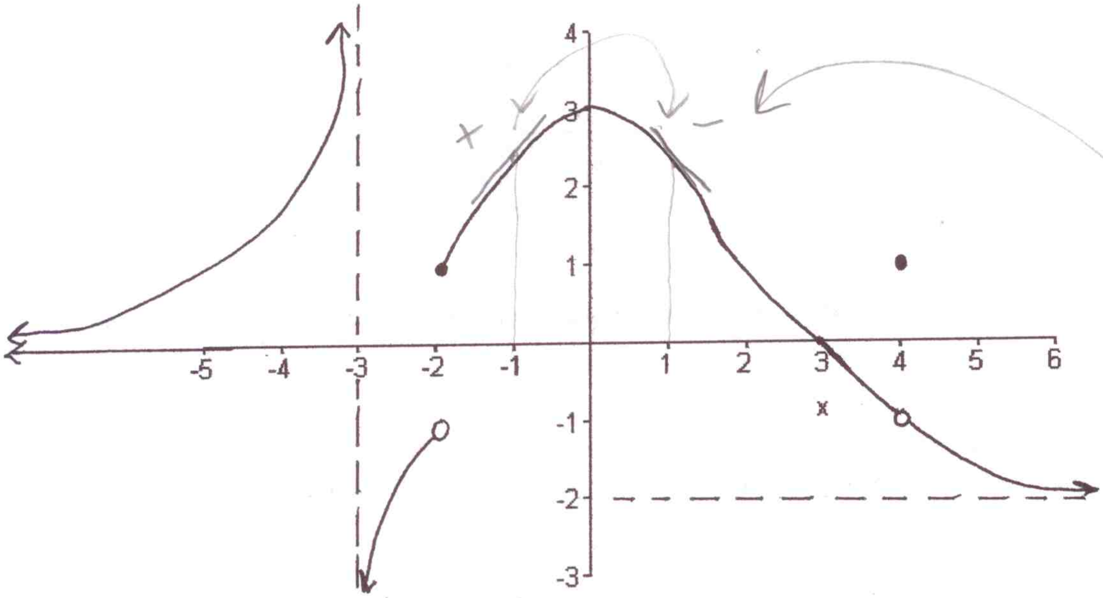
9/25/2006
Dr. Lunsford

MA 261 Calculus I
Test 1

Name: Solution
(~~100~~ 112 Points Total)

For all limits on this test, determine if the limit exists as a number, exists in the infinite sense, or does not exist. If the limit exists find its value.

I. Use the graph of the function f below to answer the following questions. (3 points each – ~~27~~ 39 total)



$f(0) = \underline{3}$ $f(3) = \underline{0}$ $\lim_{x \to -3^+} f(x) = \underline{-\infty}$ $\lim_{x \to 4^+} f(x) = \underline{-1}$

$\lim_{x \to -2^-} f(x) = \underline{-1}$ $\lim_{x \to -2} f(x) = \underline{DNE}$ $\lim_{x \to \infty} f(x) = \underline{-2}$ $\lim_{x \to -\infty} f(x) = \underline{0}$

$\lim_{x \to 3^+} \dfrac{\overset{③}{x}}{\underset{⓪}{f(x)}} = \underline{-\infty}$

For the remaining questions, please write "true" or "false", according to which is correct about the statement, in the space provided next to each statement.

False f is continuous at $x = 4$.   True f is continuous from the right at $x = -2$.

True f is continuous on the interval $[-2, 4)$ True $f'(-1) > f'(1)$

$\underset{+}{\smile}$ $\underset{-}{\smile}$ ← slopes of tangent lines!

II. Determine if the function given by $f(x) = \begin{cases} x^2+1, & x<0 \\ x^2+2, & 0 \le x \le 2 \\ 3x, & x>2 \end{cases}$ is continuous at $x = 2$. You must clearly justify your answer using the definition of continuity at a point. (5 points)

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 3x = 6$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2+2 = 6$

$\therefore \lim_{x \to 2} f(x) = 6$

$f(2) = 6$

$\therefore$ since $\lim_{x \to 2} f(x) = f(2)$ then f is continuous at $x = 2$

III. Find the indicated limits. *You do not need to show any intermediate steps for these problems.* <u>Very little partial credit will be given for these problems</u>. (4 points each, 16 total)

1. $\lim_{x\to 2^+}\frac{\overset{2}{x}}{\underset{0^-}{2-x}} = -\infty$

2. $\lim_{x\to\infty}\frac{7-8x^2}{3x^2+11} = -\frac{8}{3}$

3. $\lim_{x\to -1} 4x^{13}-9x^{15}+11x^{14} = 16$

$-4+9+11$

4. $\lim_{x\to -\infty} 4x^{13}\boxed{-9x^{15}}+11x^{14} = +\infty$

$-9(-\infty)$

IV. Consider the following table of values given below. Please answer the following questions (3 points each, 6 total)

x	1.9	1.99	1.995	2.005	2.01	2.1
$f(x)$	-4.013	-4.006	-4.002	-3.997	-3.991	-3.839

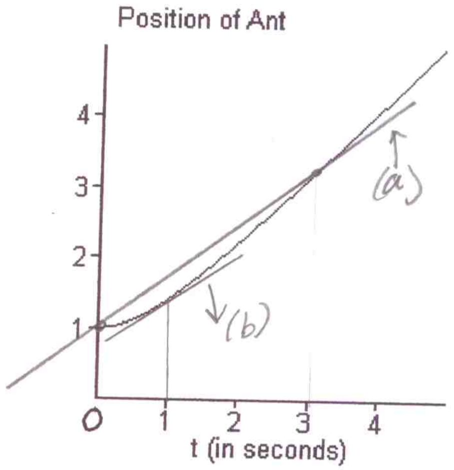
(a) From the numerical evidence above, what is $\lim_{x\to 2} f(x)$?

-4

(b) Is it possible that $\lim_{x\to 2} f(x)$ equals some other number than your answer in part (a)? Why or why not?

Yes. We can get much closer to 2 and anything could happen with f(x) in the interval (1.995, 2.005).

V. An ant walks along a ruler. The ant's position is given by the function $p(t)=\sqrt{t^2+1}$ where $p(t)$ is in inches, t is in seconds. A graph of this position function is given below. Please answer the following questions. (22 points total)



Position of Ant

(a) Find the average velocity of the ant from time $t=0$ to $t=3$ seconds. Draw and clearly indicate the line on the graph whose slope represents this velocity. (5 points)

$\frac{\Delta p}{\Delta t} = \frac{p(3)-p(0)}{3-0} = \frac{\sqrt{10}-1}{3} \approx .7208$ in/s

(b) Find the velocity of the ant at time $t=1$ seconds. Draw and clearly indicate the line on the graph whose slope represents this velocity. (8 points)

$\lim_{\Delta t\to 0}\frac{\Delta p}{\Delta t} = \lim_{t\to 1}\frac{p(t)-p(1)}{t-1} = \lim_{t\to 1}\frac{\sqrt{t^2+1}-\sqrt{2}}{t-1}$

$= \lim_{t\to 1}\frac{t^2+1-2}{(t-1)(\sqrt{t^2+1}+\sqrt{2})} = \lim_{t\to 1}\frac{t^2-1}{(t-1)(\sqrt{t^2+1}+\sqrt{2})} = \lim_{t\to 1}\frac{t+1}{\sqrt{t^2+1}+\sqrt{2}}$

$= \frac{2}{2\sqrt{2}} = \boxed{\frac{1}{\sqrt{2}}\text{ in/s}}$

(c) Find the velocity of the ant at time $t=a$ seconds. (9 points)

$\lim_{t\to a}\frac{p(t)-p(a)}{t-a} = \lim_{t\to a}\frac{\sqrt{t^2+1}-\sqrt{a^2+1}}{t-a} = \lim_{t\to a}\frac{t^2+1-(a^2+1)}{(t-a)(\sqrt{t^2+1}+\sqrt{a^2+1})}$

$= \lim_{t\to a}\frac{t+a}{\sqrt{t^2+1}+\sqrt{a^2+1}} = \frac{2a}{2\sqrt{a^2+1}} = \frac{a}{\sqrt{a^2+1}}$

Note: When a=1 we get the answer for part (b)!

VI. Find the indicated limits. You must show at least one intermediate step to receive full credit. (6 points each – 24 points total) $\nearrow$ -3

1. $\lim_{u\to -2^-}\frac{u^2+u-2}{u^2+4u+4} = \lim_{u\to -2^-}\frac{(u+2)(u-1)}{(u+2)^2} = \lim_{u\to -2^-}\frac{u-1}{\underset{\searrow 0^-}{u+2}} = +\infty$

If plug & chug get $\frac{0}{0}$ indet. form

$u\to -2^-$
$\downarrow$
$u<-2$
$u+2<0 \rightarrow 0^-$

2. $\lim_{t\to 3}\frac{\frac{1}{t^2+1}-\frac{1}{10}}{t-3} = \lim_{t\to 3}\frac{10-(t^2+1)}{(t-3)(10)(t^2+1)}$

$= \lim_{t\to 3}\frac{9-t^2}{(t-3)(10)(t^2+1)} = \lim_{t\to 3}\frac{-(3+t)}{10(t^2+1)}$

$= \frac{-6}{100} = \boxed{\frac{-3}{50}}$

If plug & chug get $\frac{0}{0}$ indet form

3. $\lim_{x\to -1^+}\frac{x^2+3x+2}{2x^2+x-1} = \lim_{x\to -1^+}\frac{(x+2)(x+1)}{(2x-1)(x+1)}$

$= \lim_{x\to -1^+}\frac{\overset{\nearrow 1}{x+2}}{\underset{\searrow -3}{2x-1}} = \boxed{-\frac{1}{3}}$

✓ If plug & chug get $\frac{0}{0}$ Indet. form

4. $\lim_{x\to -\infty}\frac{\overset{\nearrow -\infty}{x^3-x+1}}{\underset{\downarrow -\infty}{3-x^2}} = \lim_{x\to -\infty}\frac{\overset{\nearrow 1}{1-\frac{1}{x^2}+\frac{1}{x^3}}}{\underset{\searrow 0^+}{\frac{3}{x^3}-\frac{1}{x}}} = +\infty$

Get $\frac{-\infty}{-\infty}$ Indet. form

$\lim_{x\to -\infty}\frac{\overset{(-)}{3-x^2}}{\underset{(-)}{x^3}} = 0^+$

Note: Positive sign for ratio!