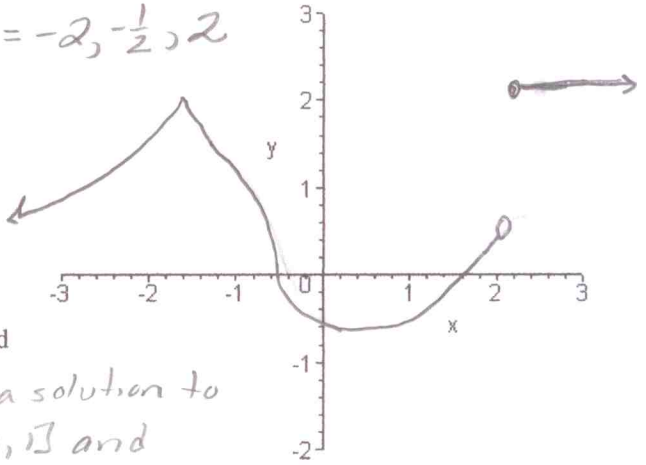
Pledge:

2/16/2009 MATH261 Calculus I Name: Solution
Dr. Lunsford Quiz 5 (20 Points Total)

I. Use the graph of the function f given below to answer the following. (4 points total)



1. List *all values* of x at which f is NOT differentiable. (2 points) $x = -2, -\frac{1}{2}, 2$

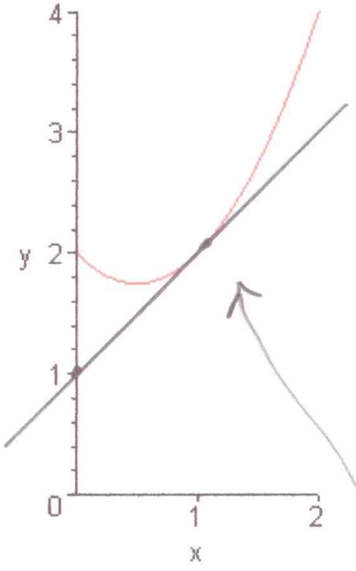
2. True or False (circle one): $f'(-3) > f'(-1)$. (1 point)

3. True or False (circle one): $f'(1) < f'(-1)$. (1 point)

II. Explain, using the Intermediate Value Theorem (IVT), why the equation $e^{-x^2} = x$ has a solution on the interval $[0,1]$. Hint: First define a function to which you can apply the IVT. DO NOT try to find the solution. (4 points)

Let $f(x) = e^{-x^2} - x$. If $f(x) = 0$ then x is a solution to the equation. Clearly f is continuous on $[0,1]$ and $f(0) = 1 - 0 = 1 > 0$ and $f(1) = e^{-1} - 1 < 0$. Thus by the IVT there exists $c \in [0,1]$ such that $f(c) = 0$. c is a solution to the equation.

III. The displacement of an object (in meters) moving along a straight path is given by the function $f(x) = 2 - x + x^2$ where x is given in seconds. To your right you are given a graph of this function. (12 points total)



(a) USE THE DEFINITION OF THE DERIVATIVE FUNCTION to show that $f'(x) = -1 + 2x$. NEATLY show all of your work. (8 points)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2 - (x+h) + (x+h)^2 - (2 - x + x^2)}{h}$$

$$= \lim_{h \to 0} \frac{-h + 2xh + h^2}{h} = \lim_{h \to 0} -1 + 2x + h$$

$$= -1 + 2x$$

(b) Find the velocity of the object at $x = 1$ second. (1 point) $f'(1) = -1 + 2(1) = 1 \text{ m/s}$

(c) Find the equation of the line whose slope represents the velocity found in part (b). Accurately graph this line on the graph of f given above. (3 points)

$y - 2 = 1(x - 1) \Rightarrow y = x + 1$