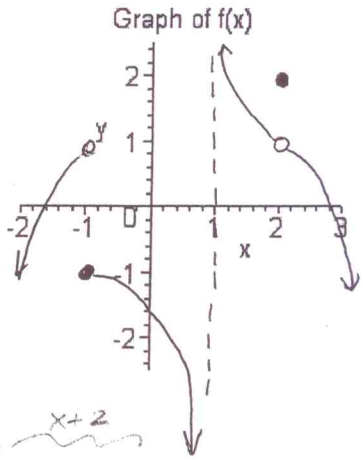
Pledge:

9/21/2006 MATH261 Calculus I Name: Solution
Dr. Lunsford Quiz 5 (20 Points Total)

For any limits on this quiz, find the limit if it exists. If the limit does not exist as a number, determine if it exists in the infinite sense (i.e. determine if it is ∞ or $-\infty$) or just does not exist.

I. Use the graph of the function f to answer the following. (6 points total)



1. $\lim_{x\to 1^-} f(x) = -\infty$ (1 point)
2. List all real numbers at which f is not continuous. (2 points)

$-1, 1, 2$

3. List all real numbers at which f has a removable discontinuity. (1 point)

2

4. (True) or False (circle one): f is continuous on the interval $[-1,1)$
5. True or (False) (circle one): f is continuous on the interval $(1,2]$

II. Consider the function $f(x)=\begin{cases} x-3, & x<-1 \\ x^2-4, & -1\le x<2 \\ x+2, & 2\le x \end{cases}$. For each value of x given below, determine if f is continuous at that value. You must justify your answer using the definition of continuity at a point. (4 points each, 8 points total)

$x-3$ $\quad x^2-4$ $\quad x+2$

$-1 \quad 0 \quad 2$

f is cont. @ $x=a \iff \lim_{x\to a} f(x) = f(a)$

1. $x=2$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x+2 = 4$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2-4 = 0$

$\therefore \lim_{x\to 2} f(x)$ DNE $\Rightarrow f$ is not continuous at $x=2$.

2. $x=0$

$\lim_{x\to 0} f(x) = \lim_{x\to 0} x^2-4 = -4$

$f(0) = -4$

$\therefore$ since $\lim_{x\to 0} f(x) = f(0)$, f is continuous at $x=0$.

III. Evaluate the following limits. For each limit you must show at least one intermediate step for full credit. (3 points each, 6 points total)

1. $\lim_{w\to 2^+} \frac{2-w}{w^2-4w+4}$ $\quad (0)$

$\frac{0}{0}$ Indeterminate form

$= \lim_{w\to 2^+} \frac{2-w}{(w-2)^2}$ $\quad (-1)$

$= \lim_{w\to 2^+} \frac{-1}{w-2} = -\infty$ $\quad (0^+)$

$\frac{-1}{0^+} \to \infty$

2. $\lim_{x\to -4^-} \frac{x+3}{x+4}$ $\quad (-1)$

$= +\infty$ $\quad (0^-)$

as $x\to -4^-$ $\quad x+4 \to 0^-$

$x<-4 \Rightarrow x+4<0$