2/15/2006 MATH261 Calculus I Name: Solution

Dr. Lunsford Quiz 4 (20 Points Total)

You must show all work on this quiz for full credit.

Definition: f is continuous at $x=2 \iff \lim_{x\to 2} f(x) = f(2)$

1. Given the function $f(x)=\begin{cases} \dfrac{x^2-2x}{4-x^2}, & x<2 \\ -\dfrac{1}{2}, & x\ge 2 \end{cases}$ determine if f is continuous at $x=2$. (4 points)

$f(2) = -\frac{1}{2}$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}\left(-\frac{1}{2}\right) = -\frac{1}{2}$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \dfrac{x^2-2x}{4-x^2} = \lim_{x\to 2^-} \dfrac{x(x-2)}{(2-x)(2+x)} = \lim_{x\to 2^-} \dfrac{-x}{2+x} = \dfrac{-2}{4} = -\dfrac{1}{2}$

$\frac{0}{0}$

Thus $\lim_{x\to 2} f(x) = -\frac{1}{2} = f(2)$ $\therefore$ f is continuous at $x=2$.

2. Use the function $f(x) = \dfrac{x-2}{x^2-5x+6}$ to answer the following questions (7 points total):

Since $x^2-5x+6 = (x-2)(x-3)$, $x=2$ and $x=3$ are candidates for vert. asymp.

(a) Find the equation of all vertical asymptotes of f. Clearly indicate your answers. (4 points)

$\lim_{x\to 2} f(x) = \lim_{x\to 2} \dfrac{x-2}{(x-2)(x-3)} = \lim_{x\to 2} \dfrac{1}{x-3} = -1$ $\therefore$ $x=2$ is not a v.a.

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \dfrac{1}{x-3} = +\infty \Rightarrow$ $\boxed{x=3 \text{ is a v.a.}}$

(b) Find all horizontal asymptotes of f. Clearly indicate your answers. (3 points)

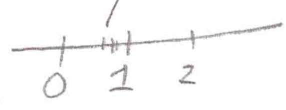
$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{x-2}{x^2-5x+6} = 0$

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \dfrac{x-2}{x^2-5x+6} = 0$

$\boxed{\text{Thus } y=0 \text{ is the only h.a. for } f.}$

3. Find the indicated limits. If the limit DNE as a number please determine if it exists in the infinite sense. (3 points each, 9 total)

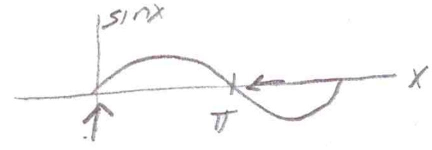
$x<1 \Rightarrow x-1<0$



(a) $\lim_{w\to 1^-} \dfrac{w-2}{w-1} = \boxed{+\infty}$

-1

0^-



(b) $\lim_{\theta\to\pi^+} \dfrac{\theta}{\sin(\theta)} = \boxed{-\infty}$

π

0^-

(c) $\lim_{x\to-\infty} 3x^3 - 4x^6 - 11x^7 = \boxed{\infty}$

leading term

$-11(-\infty) = \infty$

OR $\lim_{x\to-\infty} (3x^3-4x^6-11x^7)$

$= \lim_{x\to-\infty} x^7\left(\frac{3}{x^4} - \frac{4}{x} - 11\right) = +\infty$

$-\infty$ (-11)