Pledge:

2/3/2009 MATH261 Calculus I Name: Solution
Dr. Lunsford Quiz 3 (30 Points Total)

I. Basic limits. Find the exact value of the following limits (i.e. not a calculator approximation). You are not required to show an intermediate step for these problems. (2 points each – no partial credit, 8 points total)

1. $\lim_{t\to\infty}\left(10^7 t^3 \underbrace{-11t^4}_{(-11)(+\infty)} + 10^6 t^2\right) = \boxed{-\infty}$ leading term

2. $\lim_{x\to 1^+}\frac{x-2}{x-1} = \boxed{-\infty}$ $\quad$ ($x-2 \to -1$, $x-1 \to 0^+$)

3. $\lim_{x\to\infty}\frac{3x-4x^3}{6+2x^3} = \lim_{x\to\infty}\frac{\frac{3}{x^2}-4}{\frac{6}{x^3}+2} = \boxed{-2}$

4. $\lim_{x\to-\infty}\frac{x-1}{x^3+7} = \lim_{x\to-\infty}\frac{\frac{1}{x^2}-\frac{1}{x^3}}{1+7} = \boxed{0}$

II. More interesting limits. Find the exact value of the following limits (i.e. not a calculator approximation). For each limit you must show at least one intermediate step for full credit. (5 points each, 10 points total)

1. $\lim_{w\to 4^-}\frac{-8+6w-w^2}{w^2-8w+16} = \lim_{w\to 4^-}\frac{-(w-4)(w+2)}{(w-4)(w-4)} = \lim_{w\to 4^-}\frac{-(w+2)}{w-4} = \boxed{+\infty}$ $\quad$ ($-(w+2)\to -2$, $w-4 \to 0^-$)

$\frac{0}{0}$ indet form

2. $\lim_{x\to-\infty}\frac{4x+7}{\sqrt{9x^2+x-4}} \overset{IBL}{=} \lim_{x\to-\infty}\frac{4x}{\sqrt{9x^2}} = \lim_{x\to-\infty}\frac{4x}{3|x|} = \lim_{x\to-\infty}\frac{4x}{3(-x)}$

$\frac{\infty}{\infty}$ Indet form.

$= \lim_{x\to-\infty}\frac{-4}{3} = \boxed{-\frac{4}{3}}$ also see Example 6 page 135

III. Let $f(x)=\begin{cases}\frac{1}{x+2} & ,x<-2\\ x & ,-2\le x<2\\ \frac{x^2}{2} & ,2\le x\end{cases}$. Please answer the following questions using this function. (4 points each, 12 total)

To be continuous @ $x=2$ we must show $\lim_{x\to 2} f(x) = f(2)$

(a) Is f continuous at $x=2$? You must show all work to justify your answer.

$\lim_{x\to 2} f(x) = ?$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}\frac{x^2}{2} = 2$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x = 2$

$\therefore \lim_{x\to 2} f(x) = 2$

$f(2) = \frac{2^2}{2} = 2$

$\therefore \lim_{x\to 2} f(x) = 2 = f(2)$

So f is cont. @ $x=2$.

(b) Is $x=-2$ a vertical asymptote of the function? Again show all work to justify your answer.

$\lim_{x\to -2^-} f(x) = \lim_{x\to -2}\frac{1}{x+2} = -\infty$ $\quad$ ($x+2 \to 0^-$) $\to$ Thus $x=-2$ is a v.a. of f

(c) Does f have a horizontal asymptote? Why or why not? Please show all work to justify your answer.

Need to show: either $\lim_{x\to\infty} f(x) = c$ or $\lim_{x\to-\infty} f(x) = c$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{x^2}{2} = \infty$ $\therefore$ no horz asymp as $x\to\infty$

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\frac{1}{x+2} = 0$ $\therefore$ $y=0$ is a horz asymp. of f.