Pledge:

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Dr. Lunsford

MATH261 Calculus I
Quiz 2

Name: Solution
(20 Points Total)

I. Basic limits. Find the exact value of the following limits (i.e. not a calculator approximation). You are not required to show an intermediate step for these problems. (1 points each, 2 points total)

1. $\lim_{t\to 2^+} \sqrt{2-t} = \boxed{DNE}$

$t \to 2^+ \quad 2-t \to 0^-$

2. $\lim_{t\to 1} \left(t^2+1\right)^3 \left(t+3\right)^2 = 2^3 4^2 = \boxed{128}$

II. More interesting limits. Find the exact value of the following limits (i.e. not a calculator approximation). For each limit you must show at least one intermediate step for full credit. (18 points total)

1. $\lim_{w\to 2^+} \frac{w-2}{8-w^3} = \lim_{w\to 2^+} \frac{w-2}{(2-w)(4+2w+w^2)} = \lim_{w\to 2^+} \frac{-1}{4+2w+w^2}$

(3 points) $\downarrow$ $\frac{0}{0}$ Indet Form

$= \frac{-1}{4+4+4} = \boxed{\frac{-1}{12}}$

2. $\lim_{x\to 0} \left[\frac{x}{\sqrt{2+x}-\sqrt{2}} \cdot \frac{(\sqrt{2+x}+\sqrt{2})}{(\sqrt{2+x}+\sqrt{2})}\right] = \lim_{x\to 0} \frac{x\,(\sqrt{2+x}+\sqrt{2})}{2+x-2}$

(4 points) $\downarrow$ $\frac{0}{0}$ Indet Form

$= \lim_{x\to 0} \sqrt{2+x}+\sqrt{2} = \sqrt{2}+\sqrt{2} = \boxed{2\sqrt{2}}$

3. $\lim_{h\to 0} \frac{(4+h)^{-1}-4^{-1}}{h} = \lim_{h\to 0} \frac{\left(\frac{1}{4+h}-\frac{1}{4}\right)}{(h)} \cdot \frac{(4(4+h))}{(4(4+h))} =$

(4 points) $\downarrow$ $\frac{0}{0}$ Indet

$= \lim_{h\to 0} \frac{4-(4+h)}{4h(4+h)} = \lim_{h\to 0} \frac{-h}{4h(4+h)}$

$= \lim_{h\to 0} \frac{-1}{4(4+h)} = \boxed{-\frac{1}{16}}$

4. $\lim_{t\to 3} \sqrt{t^2+2t-10} = \lim_{u\to 5} \sqrt{u} = \boxed{\sqrt{5}}$

To receive full credit for problem number 4 you must explicitly show a substitution and change of variable in the limit, i.e. let $u = \cdots$. (3 points)

Let $u = t^2+2t-10$. As $t\to 3,\ u\to 5$ i.e. $\lim_{t\to 3} u = \lim_{t\to 3} t^2+2t-10 = 5$

5. $\lim_{x\to -6^-} \frac{2x+12}{|x+6|} = \lim_{x\to -6^-} \frac{2(x+6)}{-(x+6)} = \lim_{x\to -6^-} -2 = \boxed{-2}$

$0/0$ Indet form

(4 points)

$|x+6| = \begin{cases} x+6, & x+6 \geq 0 \\ -(x+6), & x+6 < 0 \end{cases} \iff |x+6| = \begin{cases} x+6, & x \geq -6 \\ -(x+6), & x < -6 \end{cases}$