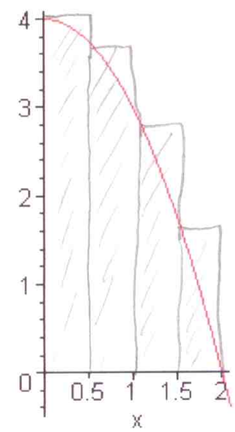
Pledge:

4/14/2009 MATH261 Calculus I Name: Solution

Dr. Lunsford Quiz 12 (40 Points Total)

I. Below you are given the graph of $y = 4 - x^2$. Please answer the following questions. (7 points total)

Graph for Problem I



(a) Compute the left endpoint sum from $x = 0$ to $x = 2$ with $n = 4$ equal length sub-intervals. You DO NOT need to simplify your answer (i.e. once you have all numbers - STOP!) Represent this sum graphically using the graph to your right. (5 points)

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

$\frac{1}{2}\left[f(0) + f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2})\right] =$

$\boxed{2 + \frac{15}{8} + \frac{3}{2} + \frac{7}{8}} = 6.25$

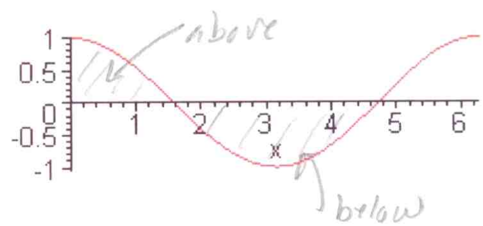
(b) The sum computed in part (a) is (circle one)

more than less than equal to

the integral $\int_0^2 4 - x^2 dx$. (2 points) clear from shaded area

II. Below you are given the graph of the cosine function. Please answer the following questions. (6 points total)

Graph for Problem II



(a) Find $\int_0^{3\pi/2} \cos(x)dx$. (4 points)

$= \sin x \Big]_{x=0}^{3\pi/2} = \sin \frac{3\pi}{2} - \sin 0$

$= -1 - 0 = \boxed{-1}$

(b) Explain using the graph of the cosine function why your answer in part (a) is a negative number. (2 points)

More area between the curve and the x-axis lies below the x-axis. Since this below the x-axis area is computed as a negative number it makes sense that the integral is negative

III. Suppose $g(x) = \int_0^{x^2} (1+t^3)^{100} dt$. Find $g'(x)$. (5 points)

Let $F(x)$ be an antiderivative of $(1+x^3)^{100}$. Then

$F'(x) = (1+x^3)^{100}$

Now $g(x) = F(x^2) - F(0)$ so $g'(x) = \frac{d}{dx}\left[F(x^2) - F(0)\right]$

$= F'(x^2) \cdot 2x = (1+(x^2)^3)^{100}(2x) = (1+x^6)^{100}(2x)$

Pledge:

IV. Find the indicated integrals. You should show at least one intermediate step. (4 points each – 12 points total)

1. $\int_0^1 \frac{4}{t^2+1}\,dt = 4\arctan t\Big]_{t=0}^{t=1} = 4\arctan(1) - 4\arctan(0)$

$= 4\left(\frac{\pi}{4}\right) - 0 = \boxed{\pi}$

2. $\int_1^9 \frac{1}{2w}\,dw = \frac{1}{2}\int_1^9 \frac{1}{w}\,dw = \frac{1}{2}\ln|w|\Big]_{w=1}^{9} = \frac{1}{2}\ln 9 - \frac{1}{2}\ln 1$

$\boxed{= \frac{1}{2}\ln 9 = \ln 3}$

3. $\int_1^8 \sqrt[3]{x}\,dx = \int_1^8 x^{1/3}\,dx = \frac{3}{4}x^{4/3}\Big]_{x=1}^{8} = \frac{3}{4}\left[8^{4/3}\right] - \frac{3}{4}(1)$

$= \frac{3}{4}(16) - \frac{3}{4} = 12 - \frac{3}{4} = \boxed{\frac{45}{4}}$

V. Find the indicated integrals. You must clearly show any substitutions necessary to find these integrals. (5 points each, 10 total)

1. $\int x^2 e^{x^3}\,dx = \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3} + C$

Let $u = x^3$

$du = 3x^2\,dx$

$\Rightarrow \frac{1}{3}du = x^2\,dx$

OR Let $u = e^{x^3}$

$du = e^{x^3} \cdot 3x^2\,dx$

$\Rightarrow \frac{1}{3}du = x^2 e^{x^3}\,dx$

$\frac{1}{3}\int du$

$= \frac{1}{3}u + C$

$= \frac{1}{3}e^{x^3} + C$

2. $\int x^3\left(5-x^4\right)^{10}\,dx = -\frac{1}{4}\int u^{10}\,du$

Let $u = 5 - x^4$

$du = -4x^3\,dx$

$-\frac{1}{4}du = x^3\,dx$

$= -\frac{1}{4}\cdot\frac{1}{11}u^{11} + C$

$\boxed{= -\frac{1}{44}\left(5-x^4\right)^{11} + C}$