Pledge:

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Dr. Lunsford

MATH261 Calculus I
Quiz 11

Name: Solution
(20 Points Total)

I. Find the indicated integrals. (4 points each, 8 total)

(a) $\int \dfrac{3x^3 - 7\sqrt[4]{x^3} + 9x^2}{x^3}\,dx = \int (3x^3 - 7x^{3/4} + 9x^2)x^{-3}\,dx = \int 3 - 7x^{-9/4} + 9x^{-1}\,dx$

$= 3x - 7\left(-\tfrac{4}{5}x^{-5/4}\right) + 9\ln|x| + C$

$= \boxed{3x + \dfrac{28}{5}x^{-5/4} + 9\ln|x| + C}$

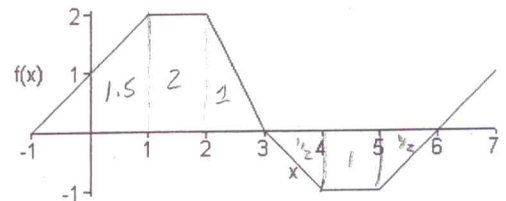(b) $\int 3e^x - \dfrac{2}{1+x^2}\,dx = \boxed{3e^x - 2\arctan x + C}$

II. Below you are given the graph of a function $f(x)$. The graph consists entirely of straight line segments. Please find the indicated definite integrals. Note that you should be able to use basic geometry to find all of these integrals (i.e. you do not need to know the function rule for f). (1 point each, 3 total)

Graph of f(x)

(a) $\int_1^3 f(x)\,dx = \boxed{3}$

(b) $\int_3^5 f(x)\,dx = \boxed{-1.5}$

(c) $\int_0^6 f(x)\,dx = 4.5 - 2 = \boxed{2.5}$

III. A particle moves up (positive direction) and down (negative direction) the y axis with velocity given by $\dfrac{dy}{dt} = v(t) = \sin(t) - \cos(t)$ in units per second. To your right you are given a graph of this velocity function. Please answer the following questions. (9 points total)

Velocity Function

(a) Find the change in position of the particle from time $t = 0$ to $t = \pi$ seconds. (4 points)

$\int_0^\pi \sin(t) - \cos(t)\,dt = -\cos t - \sin t \Big|_{t=0}^{\pi}$

$-\cos \pi - \sin \pi - (-\cos 0 - \sin 0) = 1 + 1 = \boxed{2 \text{ units}}$

(b) If the particle is at the one unit mark at $t = 0$ seconds, find a function that gives the position of the particle. (4 points)

$p(t) = \int \sin(t) - \cos(t)\,dt = -\cos t - \sin t + C \qquad \boxed{p(t) = -\cos t - \sin t + 2}$

$\Rightarrow p(0) = 1 \Rightarrow -\cos 0 - \sin 0 + C = 1 \Rightarrow -1 + C = 1 \Rightarrow C = 2$

(c) Use the graph to estimate at what time (between zero and six seconds) the particle goes from moving up the y axis to moving down the y axis. (1 point)

$\llcorner$ positive velocity $\quad \longrightarrow$ negative velocity $\qquad \boxed{\approx 4 \text{ seconds}}$

BONUS: Find the exact time asked for in part (c). Please show all work. (2 points)

$v(t) = 0 \Rightarrow \sin(t) - \cos(t) = 0 \quad \Rightarrow \quad t = \tfrac{\pi}{4} \text{ or } t = \tfrac{\pi}{4} + \pi \quad (t \text{ between } 0 \text{ and } 6 \text{ sec})$

$\Rightarrow \sin t = \cos t$

$\Rightarrow \tan t = 1 \qquad \boxed{t = \tfrac{5\pi}{4} \approx 3.927 \text{ seconds}}$