

Pledge:

11/10/2006
Dr. Lunsford

MATH261 Calculus I
Quiz 11

Name: Solution
(20 Points Total)

I. Below you are given the graph of the function $f(x) = x^3(x-2)^4$. Please answer the following questions: (17 points total)

(a) Find the x coordinates of all critical points of the function. (5 points)

$$\begin{aligned} f'(x) &= 3x^2(x-2)^4 + x^3(4(x-2)^3(1)) \rightarrow f'(x) = 0 \\ &= x^2(x-2)^3 [3(x-2) + 4x] \\ &= x^2(x-2)^3 [7x-6] \end{aligned}$$

$\Rightarrow \begin{aligned} x &= 0 \\ x &= 2 \\ x &= \frac{6}{7} \end{aligned}$

(b) Construct a first derivative sign chart for the function. (5 points)

Intervals of f'	$(-\infty, 0)$	$(0, \frac{6}{7})$	$(\frac{6}{7}, 2)$	$(2, \infty)$
$x-2$	-	-	-	+
$7x-6$	-	-	+	+
sign f'	+	+	-	+

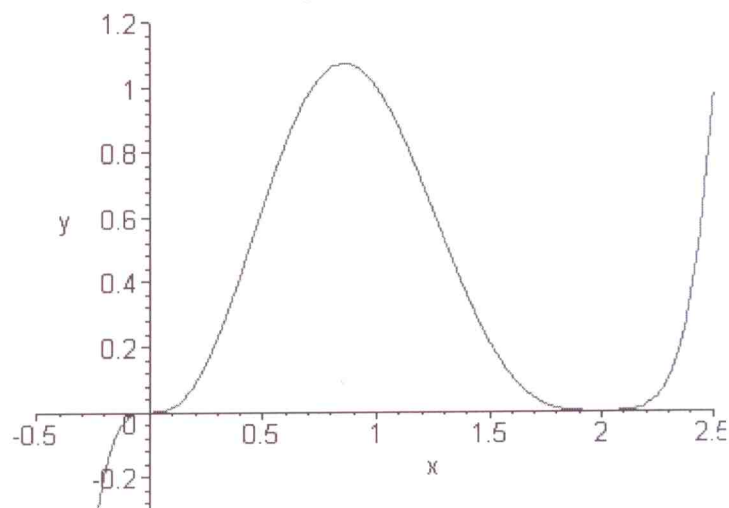
(c) At what values of x does the function have relative extrema? Indicate whether the extreme value is a relative maximum or minimum. Justify your answers based on the sign chart you found in part (b). (4 points)

f' changes from + to - @ $x = \frac{6}{7}$ $\therefore$ there is a rel. max value @ $x = \frac{6}{7}$
 f' changes from - to + @ $x = 2$ $\therefore$ there is a rel. min value @ $x = 2$.

(d) The second derivative of the function is $f''(x) = 6x(x-2)^2(7x^2 - 12x + 4)$. Use the second derivative test to determine if the critical points found in part (a) give relative extrema for the function. Indicate why the extreme value is a relative maximum or minimum or why the second derivative test is inconclusive. (3 points)

$$\begin{aligned} f''(0) &= 0 \text{ inconclusive} \\ f''(2) &= 0 \text{ inconclusive} \\ f''(\frac{6}{7}) &< 0 \Rightarrow \text{there is a rel.} \\ &\quad \text{max @ } x = \frac{6}{7} \end{aligned}$$

Graph for Problem I



II. Below you are given the graph of the first derivative of a function. Please answer the following questions. (3 points total)

(a) At what values of x does the function have critical points? (1 point)

$f'(x) = 0$ when $x = -1$ and $x = 2$

(b) On what interval(s) is the function decreasing? (1 point)

$f'(x) < 0$ on $(-1, 2)$

(c) Circle the statement below that is the most correct: (1 point)

- The function has a relative maximum value at $x = 2$.
- The function has a relative minimum value at $x = 2$.
- The function does not have a relative extreme value at $x = 2$.

f' changes from $-$ to $+$ at $x = 2$

Graph of First Derivative of a Function

