Pledge:

3/31/2009 MATH261 Calculus I Name: Solution
Dr. Lunsford Quiz 10 (20 Points Total)

I. Find the indicated limits. You must show at least one intermediate step for full credit. (3 points each, 9 total)

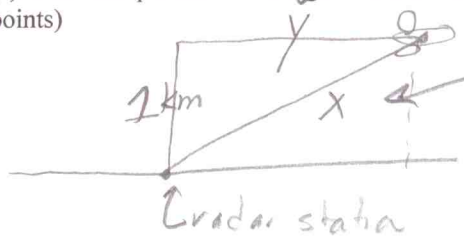
(a) $\lim_{x\to 0}\frac{x^2+\sin(3x)}{x}$ $\frac{0}{0}$ $\overset{L'H}{=}$ $\lim_{x\to 0}\frac{2x+3\cos(3x)}{1} = \boxed{3}$

(b) $\lim_{w\to\infty} w\tan\left(\frac{4}{w}\right)$ $\infty\cdot 0$ $= \lim_{w\to\infty}\frac{\tan(4w^{-1})}{w^{-1}}$ $\frac{0}{0}$ $\overset{L'H}{=}$ $\lim_{w\to\infty}\frac{\sec^2(4w^{-1})(-4w^{-2})}{-w^{-2}}$ $= \lim_{w\to\infty}\sec^2(4w^{-1})(4) = \boxed{4}$

(c) $\lim_{t\to 0}\frac{t\cos t}{e^{2t}}$ $\frac{0\cdot 1}{1} = \frac{0}{1} = \boxed{0}$

II. A plane flying horizontally above the ground with constant speed of 300 km/h passes over a ground radar station at an altitude of 1 km. At what rate is the distance from the plane to the radar station increasing one minute later? Hint: Let x be the distance from the plane to the radar station and y be the horizontal distance of the plane to the radar station (both in km). In the drawing below $x = 1$ km and $y = 0$ km. Please answer the following questions. (11 points total)

(a) Draw the picture on the right at a later time. Be sure to clearly label all variables on your picture. (2 points)



(b) Please state your goal for this problem in terms of the variables x and y. (2 points)

Find $\left.\frac{dx}{dt}\right|_{t=1 \text{ minute later}}$ $1 \text{ min} = \frac{1}{60}$ hr

(c) Please state what you are given in terms of the variables x and y. (2 points)

$\frac{dy}{dt} = 300$ km/hr

(d) Find a relationship between the variables x and y that holds for all time of interest for this problem. (2 points)

$y^2 + 1 = x^2$

(e) Solve the problem. Clearly indicate your answer. (3 points)

at $t = 1$ min
$y = 300\frac{\text{km}}{\text{hr}}\cdot\frac{1}{60}\text{ hr} = 5$ km
$\Rightarrow x^2 = 5^2 + 1^2 \Rightarrow x = \sqrt{26}$ km

$2y\frac{dy}{dt} = 2x\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{y}{x}\frac{dy}{dt}$ $\rightarrow \left.\frac{dx}{dt}\right|_{t=1\text{min}} = \boxed{\frac{5}{\sqrt{26}}(300)\text{ km/hr}}$

$\approx \boxed{294.17 \text{ km/hr}}$