

Pledge:

10/27/2006
Dr. Lunsford

MATH261 Calculus I
Quiz 10

Name: Solution
(20 Points Total)

I. Find the indicated derivatives. Do not simplify your answers. (4 points each, 8 total)

(a) $F(y) = y^4 \ln(1 + e^{4y})$, $F'(y) = 4y^3 \ln(1 + e^{4y}) + y^4 \left(\frac{1}{1 + e^{4y}} (4e^{4y}) \right)$

(b) $y = (\cos(2x))^{\ln x}$, $\frac{dy}{dx} = (\cos(2x))^{\ln x} \left[\frac{\ln(\cos(2x))}{x} - 2 \ln x + \tan(2x) \right]$

$\ln y = \ln((\cos(2x))^{\ln x})$
 $\Rightarrow \ln y = \ln x (\ln(\cos(2x)))$

$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x} \ln(\cos(2x)) + \ln x \left(\frac{1}{\cos(2x)} (-\sin(2x)(2)) \right)$

$\frac{dC}{dt} = 6 \frac{\text{in}}{\text{s}}$

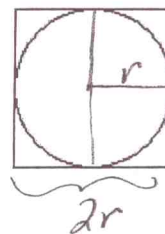
II. A circle is inscribed in a square as shown in the figure below. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency. Please answer the following. Neatly show all of your work. (6 points each, 12 points total)

1. Find the rate at which the perimeter of the square is increasing.

$C = 2\pi r$, $P = 4(2r) = 8r$,

$r = \frac{C}{2\pi} \rightarrow P = \frac{4}{\pi} C \Rightarrow \frac{dP}{dt} = \frac{4}{\pi} \frac{dC}{dt}$

$= \frac{4}{\pi} \left(6 \frac{\text{in}}{\text{s}} \right) = \frac{24}{\pi} \frac{\text{in}}{\text{s}} \approx 7.639 \frac{\text{in}}{\text{s}}$



2. At the instant when the area of the circle is 25π square inches, find the rate of change of the area enclosed between the circle and the square.

$A_s = (2r)^2 = 4r^2$ $A = A_s - A_c = 4r^2 - \pi r^2 = (4 - \pi)r^2$

$A_c = \pi r^2$

$A = \frac{(4 - \pi)}{(2\pi)^2} C^2$ $A_c = 25\pi \Rightarrow \pi r^2 = 25\pi$
 $\Rightarrow r = 5 \text{ in.}$
 $\Rightarrow C = 2\pi r = 10\pi$

$\frac{dA}{dt} = \frac{4 - \pi}{4\pi^2} 2C \frac{dC}{dt} \Rightarrow \left. \frac{dA}{dt} \right|_{A=25\pi} = \frac{4 - \pi}{4\pi^2} 2(10\pi) 6 = \frac{30(4 - \pi)}{\pi} \frac{\text{in}^2}{\text{s}}$
 $\approx 8.197 \frac{\text{in}^2}{\text{s}}$